

(11)

Complex Numbers

Definition of Notation: $i = \sqrt{-1}$; \bar{z} is the complex conjugate of z ; $|z|$ is the norm or absolute value of complex number z ; $a+bi$ is the rectangular form for a complex number; $r(\cos \theta + i \sin \theta)$ or $rcis \theta$ is the trigonometric form; $re^{i\theta}$ is the exponential form; (r, θ) is the polar form; $\text{Re}(z)$ is the real part of z ; $\text{Im}(z)$ is the imaginary part of z .

- 1) Simplify: $(5+3i)(2+i) - (4+i)(5+2i)$
 A) $-5-14i$ B) $-15+8i$ C) $-2+i$ D) $-11-2i$ E) NOTA
- 2) Find the product of all 5th roots of 32.
 A) 1 B) -2 C) 2 D) -32 E) 32
- 3) Find $(1+i)^6$
 A) -8 B) 8 C) $-8i$ D) $8i$ E) NOTA
- 4) Simplify: $(1+i)(2-i)(3+i)$.
 A) 8 B) $6+4i$ C) $4+4i$ D) $8+6i$ E) NOTA
- 5) If $a=1+\sqrt{3}i$ and $b=\cos 15^\circ + i \sin 15^\circ$, find a/b .
 A) $\sqrt{2} + \sqrt{2}i$ B) $2-2i$ C) $1+i$ D) $2(\text{cis } 15^\circ)$ E) NOTA
- 6) If $x^2 + 4x + 5 = 0$ and $x=a+bi$, then find $a^2 + b^2$.
 A) 5 B) 3 C) 0 D) -3 E) NOTA
- 7) Which of the following is not a 6th root of $1 + \sqrt{3}i$.
 A) $2^{1/6} (\cos 10^\circ + i \sin 10^\circ)$
 B) $2^{1/6} (\sin 20^\circ + i \cos 20^\circ)$
 C) $2^{1/6} (-\cos 50^\circ + i \sin 50^\circ)$
 D) $2^{1/6} (-\sin 80^\circ + i \sin 10^\circ)$
 E) $2^{1/6} (\cos 50^\circ - i \sin 50^\circ)$
- 8) Evaluate: $\sum_{k=0}^{199} i^k$
 A) -1 B) 1 C) 0 D) i E) NOTA
- 9) Find the sum of the 3 cube roots of 8.
 A) -2 B) 2 C) 4 D) $4\sqrt{3}$ E) NOTA
- 10) If $\frac{\sqrt{5-12i}}{\sqrt{5+2i}} = x+iy$, find $x+y$.
 A) $-\sqrt{7}$ B) $\sqrt{7}$ C) $-7/13$ D) $17/13$ E) $17/169$

- 11) Find the infinite sum of $\left(\frac{1+i}{2}\right) + \left(\frac{1+i}{3}\right)^2 + \left(\frac{1+i}{3}\right)^3 + \dots$
- A) $1+i$ B) i C) 1 D) $(1+i)/2$ E) $(1-i)/2$
- 12) If $|A| = 13$, $A = (x+ix) + 7$, and x is a negative real number. Find x .
- A) -12 B) -6 C) -5 D) $-60/7$ E) NOTA
- 13) Let $z = a+bi$ and $|b| > |a|$. If $z^2 = -4 + 3i$, find b/a .
- A) 2 B) $5/2$ C) 3 D) $2\sqrt{2}$ E) $3\sqrt{3}$
- 14) If $(2+i)$ and $3i$ are two points that are on a circle in the complex plane, which of the following complex numbers cannot also lie on that circle?
- A) $0+0i$ B) $1+i$ C) $7-4i$ D) $40+28i$
E) all of the points given can lie on the circle
- 15) Solve for x : $x^3 + 64 = 0$.
- A) $\{2+2\sqrt{3}i, -4, 2-2\sqrt{3}i\}$
B) $\{-2-2\sqrt{3}i, -4, -2+2\sqrt{3}i\}$
C) $\{-2+2\sqrt{3}i, 4, 2-2\sqrt{3}i\}$
D) $\{-4, 4\}$
E) $\{1-\sqrt{3}i, -2, 1+\sqrt{3}i\}$
- 16) Simplify the following:
 $(\cos 75^\circ + i \cos 15^\circ)(\cos 12^\circ + i \sin 12^\circ)(-\cos 87^\circ + i \sin 87^\circ)$
- A) 0 B) -1 C) 1 D) $(1+\sqrt{3})i/2$ E) NOTA
- 17) A solution of $\log_4(-1)$ is:
- A) i B) $-i$ C) 0 D) 1 E) 2
- 18) Find the sum of the 6 complex 7th roots of -1 .
- A) 0 B) -1 C) 1 D) $\cos(2\pi/7)$ E) NOTA
- 19) Simplify: $(e^{\frac{\pi}{4}i} + 1)^4$
- A) $-1+i(6+4\sqrt{2})$ B) $10i$ C) $1+10i$ D) $(6+4\sqrt{2})i$ E) NOTA
- 20) Evaluate: $(\sqrt{3+4i} + \sqrt{3-4i})$
- A) 0 B) 1 C) $\sqrt{2}$ D) $2\sqrt{2}$ E) NOTA
- 21) If $|z_1| = 5$, $|z_2| = 17$, and the real and imaginary parts of z_1 and z_2 are positive integers, what is a possible value of $|z_1 + z_2|$?
- A) $\sqrt{482}$ B) $\sqrt{433}$ C) A and B D) neither A nor B
E) not enough information

22) If we know that $(3+2i)$ is a solution of $x^5 - 5x^4 + 15x^3 - 45x^2 + 164x - 130 = 0$, find the sum of all real solutions of x .

- A) 0 B) 1 C) 10 D) 130 E) NOTA

23) If $z=a+bi$ and a and b are positive, how many of the following statements are always true?

- I) $|z| > a+b$
II) $z \cdot \bar{z} = a^2 + b^2$
III) $|\bar{z}| = |z|$
IV) $\text{Im}(z+\bar{z}) = 0$

- A) 0 B) 1 C) 2 D) 3 E) 4

24) If $x - \frac{1}{x} = 4i(\sin 9^\circ)(\cos 9^\circ)$, find x^5 .

- A) 1 B) i C) $\frac{\sqrt{3}}{2}(1+i)$ D) $\frac{\sqrt{3}}{2}(1-i)$ E) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

25) If $x = i^{i^{i^{i^{i^{\dots}}}}}$, find $\frac{x^i}{\ln x}$.

- A) 0 B) 1 C) e^i D) $1/\pi$ E) $2/\pi$

26) If $(a+bi) = (\cos 30^\circ + i \sin 30^\circ + \cos 60^\circ + i \sin 60^\circ)^6$, find a .

- A) 0 B) $(15\sqrt{3} + 26)i$ C) $(-15\sqrt{3} - 26)i$ D) $6i$ E) NOTA

27) If $x^2 + x + 1/x = -1$, find $x^8 + \frac{1}{x^8}$.

- A) -1 B) 0 C) 1 D) 2 E) 8

28) If $p=10i$, $q=11+12i$, and $r=x$ {where x is a real number}, what must x be for $(|q-r| + |p-r|)$ to be at a minimum value?

- A) $x < 3$ B) $3 \leq x < 4$ C) $4 \leq x < 5$ D) $5 \leq x < 6$ E) $x \geq 6$

29) Find: $\cos[\ln(2^i)]$

- A) $e/2$ B) $\ln(2)/4$ C) $3/4$ D) $7/4$ E) NOTA

30) Evaluate: $\ln[-(i \cos 2 + \sin 2)^2]$

- A) $-4i$ B) $-2i$ C) 0 D) $2i$ E) $4i$