

Practice Round
Alpha Ciphering
Mu Alpha Theta National Convention 2004

Find the sum of the absolute values of all x so that $x^2 + \frac{36}{x^2} = 20$.

Question # 1
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Find the non-zero coordinate of the y-intercept of the slant asymptote of the graph of

$$f(x) = \frac{2x^3 - 5x^2 + 7}{x^2 + 3x - 1}.$$

Question # 2
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Find c so that the dot product of the vectors $\langle 7 + c, 5 \rangle$ and $\langle 2, 13 \rangle$ is 45.

Question # 3
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The difference of the squares of two consecutive integers is 2003. What is the sum of the integers?

Question # 4
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Two ships depart from the same dock at the same time and travel along routes that form a 120° angle. One ship travels at 15 kilometers per hour and the other at 20 kilometers per hour. How far apart, in kilometers, are the ships two hours after departure?

Question # 5
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Solve for x if $\begin{vmatrix} 2004 & -2002 \\ x+2 & x \end{vmatrix} = 2000x + 5007$.

Question # 6
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The graph of $f(x) = ax + \frac{b}{x}$ contains the points $(2, 4a + b)$ and $\left(\frac{1}{2}, 2b + 10\right)$. Find $\frac{b}{a}$.

Question # 7
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If k is an integer larger than 1 and less than 1,000, what is the probability that $\log_3 k$ is an integer?

Question # 8
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Find the area enclosed by the curve defined parametrically by the equations $x(t) = 1 - \sin t$ and $y(t) = 3 \cos t$.

Question # 9
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Find the sum of all x in the interval $[0, 2\pi]$ that satisfy the equation $\cos 2x = 11 \sin x + 6$.

Question # 10
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Find the distance between the focus and the directrix of the parabola $y = 4x^2 + 8x - 5$.

Question # 11
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Find the product of the real solutions to the equation $x^5 - 3x^4 - 2x^3 - 4x^2 - 24x + 32 = 0$ if $-2i$ is a solution.

Question # 12
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Consider a series of similar triangles T_1, T_2, T_3, \dots where A_n is the area of triangle T_n and the scale factor $T_{n+1} : T_n$ is 2:3 for all $n > 1$. If A_1 is 405, find $A_1 + A_2 + A_3 + \dots$.