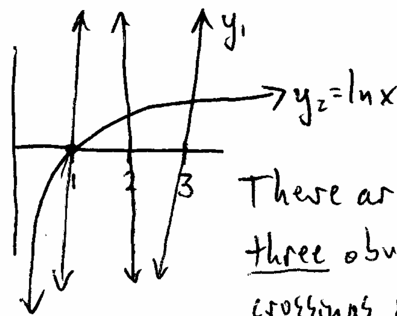
Mu Alpha Theta National Convention: Denver, 2001

Ciphering Solutions – Alpha Division

0. $\vec{A}\cdot\vec{B} = 3\cdot 12 + 4\cdot -5 = \sqrt{25}\sqrt{169}\cos\theta$

$\cos\theta = \frac{16}{65} \Rightarrow \sec\theta = \frac{65}{16}$

1.



There are ~~three~~ obvious crossings plus ~~another~~ near $x=0$, because y_1 exists for $x<0$. 4

2. $\frac{\left(-\frac{\sqrt{2}}{\cancel{2}}\right)\left(+\cancel{\sqrt{3}}\right)}{+\frac{\cancel{\sqrt{3}}}{\cancel{2}}} = -\sqrt{2}$

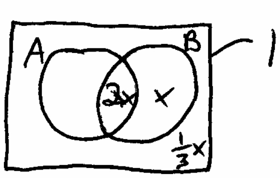
3. $r = \frac{1680}{2520} = \frac{168}{252} = \frac{42}{63} = \frac{14}{21} = \frac{2}{3}$

$S = \frac{a}{1-r} = \frac{2520}{1-\frac{2}{3}} = 7560$

4. $9 = 3\cdot 3$

$2^2\cdot 3^2 = 36$

5.



$\frac{1}{3}x = \frac{1}{10}$

$x = \frac{3}{10}$

$2x = \frac{3}{5}$

6. $\frac{1}{r} + \frac{1}{s} + \frac{1}{t} = \frac{st+rt+rs}{rst}$

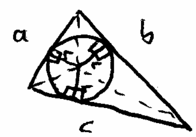
$= \frac{\frac{13}{3}}{-\frac{-42}{3}} = \frac{13}{42}$

7. $\log_{81}(\log_6(x)) = 8^{-\frac{2}{3}} = \frac{1}{4}$

$\log_6(x) = 81^{1/4} = 3$

$x = 6^3 = 216$

8.



$A = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$

$= \frac{1}{2}Pr = \frac{1}{2}\cdot 12\cdot 1$

$= 6$

9. $144 = 12^2 = 2^4 3^2$

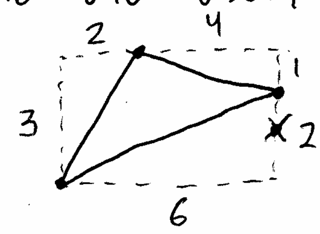
sum of factors $= (1+2+4+8+16)(1+3+9)$

$= 31\cdot 13 = 403$

10. $\sqrt{13} = \sqrt{9+4} \Rightarrow 3, 2$

$\sqrt{17} = \sqrt{16+1} \Rightarrow 4, 1$

$2\sqrt{10} = \sqrt{40} = \sqrt{36+4} \Rightarrow 6, 2$



$A = 6\cdot 3 - \frac{1}{2}(2\cdot 3 + 4\cdot 1 + 6\cdot 2)$

$= 18 - 11 = 7$

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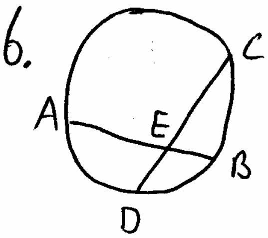
11. $\sum_{1}^{20} n^3 = \left(\frac{20\cdot 21}{2}\right)^2 = 100\cdot 441$
$= 44100$

12. $\cancel{\binom{8}{3}3!5! = 8!\ldots 8!}$
Seat boys first: $\cancel{\ldots}\ 5!\ \cancel{\ldots}$
$\cancel{\ldots} = 120$
The girls choose 3 of 6 positions between boys or on ends: $\binom{6}{3}\cdot 3!$
$= 120$
$120\cdot 120 = 14400$

13. $\sin^2 10^\circ + \sin^2 80^\circ + \sin^2 40^\circ + \sin^2 50^2$
$1 + 1 = 2$

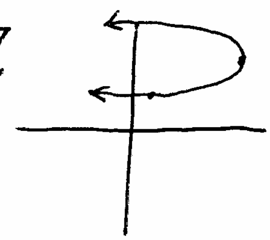
14. $0 \le x < 2\pi \Rightarrow 0 \le 2x < 4\pi$
$\sin(2x) = \frac{1}{2} \Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$
$\frac{\text{sum}}{2} = 3\pi$

15. $\frac{1}{3}\sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n+2}\right)$
$= \frac{1}{3}\left[\left(1 - \cancel{\tfrac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\tfrac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\tfrac{1}{6}}\right) + \left(\cancel{\tfrac{1}{4}} - \cancel{\tfrac{1}{7}}\right) + \cdots\right]$
$= \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{1}{3}\left(\frac{11}{6}\right) = \frac{11}{18}$

16. 

$AE\cdot EB = CE\cdot ED$

If CE or ED were 4 the other would be 8, and their product would be 32. The maximum value for the product of AE & EB is 25, so either AE or EB must be the 4, making the other one 6. 4 or 6

17. 

$x = -A(y-3)^2 + 5$
$1 = -A(1)^2 + 5$
$-4 = -A \Rightarrow A = 4$
$x = -4(y-3)^2 + 5$