

Mu Alpha Theta National Convention 2004

Alpha Applications

For each equation, NOTA means that none of the answers are correct.

1. Find the degree measure of the larger of two complimentary angles, one of which has degree measure 2 less than three times the degree measure of the other.

A) 68° B) 67° C) 23° D) 22° E) NOTA

2. A chemical storage tank is to be filled by three pipes. The first two transmit 930 L/h of liquid each and the third transmits 560 L/h. If the third pipe starts working 90 minutes after the first two begin and the tank has a capacity of 4000 L, how long (in hours) will it take after the first two pipes begin working to fill the tank?

A) 0.5 B) 1 C) 1.5 D) 2 E) NOTA

3. Individual tickets to a raffle cost \$0.35 each. A pair of consecutively numbered tickets may be bought for \$0.50. If 100 tickets were sold and \$32.20 was collected, how many pairs of consecutively numbered tickets were purchased?

A) 14 B) 28 C) 72 D) 86 E) NOTA

4. According to the theory of relativity, at high speeds the mass m of an object is

given by the equation $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ when m_0 is the mass of the object, v is the

velocity of the object, and c is the speed of light. By what factor, to the nearest

tenth, is the mass of an object increased at $\frac{2}{3}$ the speed of light?

A) 0.3 B) 0.7 C) 1.3 D) 1.7 E) NOTA

5. The ratio of the volumes of 2 rectangular boxes with square bases and equal heights is 3:4. Each side of the base of the larger box is 2 cm longer than the side of each base of the smaller. What is the area of the smaller base?

A) 36 cm^2 B) 6 cm^2 C) $\frac{9}{4} \text{ cm}^2$ D) $\frac{36}{25} \text{ cm}^2$ E) NOTA

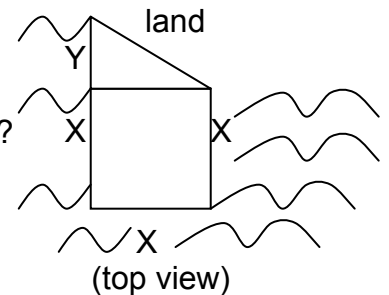
6. The limousine shuttle service operating between an airport and the center of a city charges a fare of \$10 each and carries 300 persons per day. The firm estimates that the business will decrease by 15 passengers per day for each increase of \$1 in the fare. Find the most profitable fare to charge for the service.

A) \$5 B) \$12 C) \$15 D) \$18 E) NOTA

7. The slope at which a curve in a road should be banked varies inversely with the radius of the curve and directly with the square of the maximum speed of the cars that will use it. If a curve of radius 1250 m carrying cars traveling at a maximum speed of 25 m/s has a slope of 0.049, what slope should the road have on a curve of radius 700 m if the maximum speed is 20 m/s?

- A) 0.034 B) 0.043 C) 0.056 D) 0.070 E) NOTA

8. A fishing pier is to be built in the shape of a square surmounted by a right triangle as shown in the diagram. If the area of the pier is to be 1050 m^2 and the part of its perimeter along the water is to be 100 m long, what is the length of the longest side of the pier?



- A) 40 m B) 30 m C) 20 m D) 10 m E) NOTA

9. If \$1000 is deposited in a savings account paying 6% interest compounded quarterly, how many years will it take (to the nearest year) for the money to grow to at least \$1800?

- A) 9 B) 10 C) 11 D) 12 E) NOTA

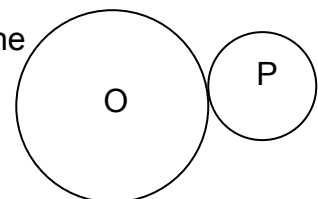
10. How many committees of 6 people can be chosen from a management group of 10 people if the president and the first vice-president of the management group are not to serve on the same committee?

- A) 28 B) 56 C) 84 D) 140 E) NOTA

11. How many distinct five-letter sequences containing exactly 3 vowels and 2 consonants can be made from the letters in the word ANTIDOTES?

- A) 3120 B) 3360 C) 3540 D) 3780 E) NOTA

12. The radius of the wheel with center O is 4 times the radius of the wheel with center P. If wheel O is stationary and wheel P rolls once around the circumference of wheel O, how many revolutions does wheel P make?



- A) 6 B) 5 C) 4 D) 3 E) NOTA

13. Will Rogers spins a lasso in a vertical circle. The diameter of the loop is 6 feet, and the loop spins 50 revolutions each minute. The lowest point on the rope remains 6 inches above the ground, and a given point, P, on the rope starts at this lowest point at time $t=0$. Which of the following is an equation that describes the height of the point, P, in feet at t seconds?

- A) $h = -3 \cos \frac{5\pi}{3} t$ B) $h = -3 \cos \frac{5\pi}{3} t + 3.5$ C) $h = -3 \cos \frac{2\pi}{3} t$
D) $h = -3 \cos \frac{2\pi}{3} t + 3.5$ E) NOTA

14. Bob is surveying a rectangle ABCD. He stands at corner A and finds the measure of the angle between side \overline{AB} and the diagonal \overline{AC} to be 30° . He then finds the measure of the angle between \overline{AC} and the line from corner A to a telephone pole on side \overline{CD} to be 45° . If the distance from corner A to corner C is 100 yards, then find the distance from corner A to the telephone pole in yards to the nearest tenth.

- A) 43.8 B) 46.1 C) 48.3 D) 51.8 E) NOTA

15. A Greek algebraist Diophantus was a boy for one-sixth of his life. After one-twelfth more he grew a beard; after one-seventh more he married; and after 5 years of marriage a son was born. The son lived half as long as the father, and Diophantus died 4 years after his son's death. How old was Diophantus when his son died?

- A) 76 B) 80 C) 84 D) 88 E) NOTA

16. A corporation has 10,000 computers, each labeled with a unique number from 1 to 10,000. What is the probability that the number of a computer chosen at random contains no 7's?

- A) $\left(\frac{9}{10}\right)^4$ B) $\left(\frac{7}{10}\right)^2$ C) $\frac{9}{10}$ D) $\left(\frac{9}{10}\right)^2$ E) NOTA

17. An ant starts at origin O in an xy-plane and stays in the first quadrant. He walks along a line that has a slope of 1 for one meter. He then turns and walks along a line with slope $\sqrt{3}$ for one meter and plants a flag at point F. What is the angle (in degrees) which the line \overline{OF} makes with the positive x-axis?

- A) 7.5° B) 52.5° C) 60° D) 62.5° E) NOTA

18. There are two red, two black, two white, and a positive but unknown number of blue socks in a drawer. It is empirically determined that if two socks are taken from the drawer without replacement, the probability they are of the same color is $\frac{1}{5}$. How many blue socks are there in the drawer?

- A) 2 B) 3 C) 4 D) 5 E) NOTA

19. Alex picks his favorite point (x, y) in the first quadrant on the unit circle $x^2 + y^2 = 1$, such that a ray from the origin through (x, y) is θ radians counterclockwise from the positive x-axis. He then computes $\cos^{-1}\left(\frac{4x + 3y}{5}\right)$ and is surprised to get θ . What is $\tan\theta$?

- A) $\frac{1}{3}$ B) $\frac{\sqrt{10}}{10}$ C) 3 D) $\sqrt{10}$ E) NOTA

20. A rectangle has sides of length $\sin x$ and $\cos x$ for some x . What is the largest possible area of such a rectangle?

- A) 0.4998 B) 0.5 C) 0.75 D) 1 E) NOTA

21. A combination lock has a 3 number combination, with each number an integer between 0 and 39 inclusive. Call the first number n_1 , the second n_2 , and the third n_3 . If you know that n_1 and n_3 leave the same remainder when divided by 4, how many possible combinations are there?

- A) 4000 B) 15600 C) 16000 D) 59319 E) NOTA

22. A ladder is leaning against a house with its lower end 15 feet from the house. When the lower end is pulled 9 feet farther from the house, the upper end slides 13 feet down. How long is the ladder (in feet)?

- A) 16 B) 18 C) 20 D) 25 E) NOTA

23. If Alex pulls a log with a force of 400 newtons at an angle of 65° with the horizontal and Kevin pulls the same log with a force of 600 newtons at an angle of 110° with the horizontal, what is the net force from Alex and Kevin on the log to the nearest whole number?

- A) 927 N B) 950 N C) 970 N D) 972 N E) NOTA

24. An airplane flies east for 210 km before turning 70° south and flying for 100 km. Find the distance of the plane from its starting point to the nearest hundredth.

- A) 261.66 km B) 120.97 km C) 14632.91 km
D) 68464.85 km E) NOTA

25. A wheel in a motor is turning counterclockwise at 2 radians per second. There is a small hole in the wheel 3 centimeters from its center. Suppose a model of the wheel is drawn on a rectangular coordinate system with the wheel centered at the origin. If the hole has initial coordinates (3, 0), what are its coordinates after t seconds?

- A) $(\cos 2t, \sin 2t)$ B) $(3\cos t, 3\sin t)$ C) $(3\cos 2t, 3\sin 2t)$
D) $(-3\cos 2t, -3\sin 2t)$ E) NOTA

26. When three integers are added two at a time, three distinct sums are formed: 32, 48, and 46. What is the sum of all three of the original integers?

- A) 63 B) 67 C) 93 D) 126 E) NOTA

27. If a 8-inch diameter circular pizza serves two students, how many students should two 12-inch diameter circular pizzas serve?

- A) 4 B) 6 C) 8 D) 9 E) NOTA

28. What is the maximum length of a word spelled with only A's and C's if A ___ A and C ___ C are not permitted and the word must start with A A?

- A) 5 B) 6 C) 7 D) 8 E) NOTA

29. A parabolic arch has a base 100 feet wide and a maximum height of 100 feet. How high is the arch at a point 20 feet from its center of the base?

- A) 64 B) 84 C) 92 D) 96 E) NOTA

30. A man was doodling on a tic-tac-toe board. If he chooses three boxes at random to doodle in, find the probability that those doodle boxes correspond to a winning path.

- A) $\frac{2}{21}$ B) $\frac{3}{8}$ C) $\frac{9}{84}$ D) $\frac{1}{14}$ E) NOTA

Tiebreaker 1:

To use a concentrated liquid household cleaner, you mix 1 part cleaner with 8 parts water. One quart of this solution will clean an area of 250 square feet. How many gallons of water are needed to clean a 30 ft. by 25 ft. floor?

Tiebreaker 2:

A dog is tied with a rope 10 meters long at the corner of a regular hexagonal walled garden with sides of 8 meters long. He is outside the hexagonal garden. What is the area of ground that the dog can cover as determined by the limits of the rope and the walled garden?