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- 7) A ship leaves port and sails 200 km with a bearing of 120° . It then changes its bearing to 45° and sails 120 km. What is its bearing from its original starting point? (Round to the nearest degree)
- A) 27° B) 43° C) 67° D) 93° E) NOTA
- 8) Suppose the value of a computer is represented by the function $V_t = V_0 e^{kt}$, where V_t is the value at time t of a computer with initial value V_0 . Let k be a constant. If a brand new Dell computer is valued at \$3500, but decreases \$1500 in value after $1\frac{3}{4}$ years, by how much more will it have decreased in value after another $1\frac{1}{2}$ years? (Round to the nearest \$10)
- A) \$730 B) \$760 C) \$1240 D) \$1270 E) NOTA
- 9) Two tennis players are engaging in a rally, and keep hitting the ball back and forth at a constant speed of 10 feet per second. Suppose the players are running towards each other, one at 2 feet per second and the other at 3 feet per second. The net is located halfway down the court lengthwise, parallel to both ends, and runs all the way across the 60 feet long court. If the players start facing each other with their line of sight perpendicular to the net, and they begin running from the ends of the court, how far will the ball have traveled (relative to the length of the court, parallel to the ground) once both players have stopped running (both at the net)?
- A) 150 feet B) 120 feet C) 90 feet D) 60 feet E) NOTA
- 10) A defective clock has two sets of hour and minute hands. One set reads 1:48, while the other reads 11:23. Imagining that both sets of hands are extended to form sectors of a circle with radius 8, find the common area shared by the smaller sector formed by the first set of hands, and by the smaller sector formed by the second set of hands.
- A) $\frac{528\pi}{45}$ B) $\frac{580\pi}{45}$ C) $\frac{1008\pi}{45}$ D) $\frac{1252\pi}{45}$ E) NOTA
- 11) A ladder with one end on the ground is to reach over a fence 8 feet high to a wall that is 1 feet behind the fence. To the nearest foot, what is the length of the shortest ladder that can be used?
- A) 9 feet B) 10 feet C) 11 feet D) 12 feet E) NOTA
- 12) Assume that each answer choice on this test has an equal probability of being correct, 250 students are taking it, and each student answers all 30 questions. What is the expected number of students that will correctly answer at least 10 of the 5-choice questions correctly? (Round to the nearest student)
- A) 9 B) 15 C) 50 D) 83 E) NOTA

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13) A force is applied horizontally to a ball at rest on one end of a 5-foot long table (surface parallel to the ground), which causes it to accelerate at $2.5 \frac{\text{feet}}{\text{second}^2}$. As soon as the ball leaves the other end of the table (lengthwise), gravity is the only force acting on the ball. If the table is 3 feet high, and the ball accelerates downward at $32 \frac{\text{feet}}{\text{second}^2}$, what horizontal distance does the ball travel before reaching the ground?

- A) $\frac{\sqrt{3}}{2}$ feet B) $\sqrt{3}$ feet C) $\frac{5\sqrt{3}}{4}$ feet D) $\frac{5\sqrt{3}}{2}$ feet E) NOTA

14) A gold ring is to be made from a cube of gold. The ring is to be a torus with a circular cross-section of diameter 5 millimeters with its center rotated about an axis a distance of 8 millimeters away. Find the length of a side of the cube. (Assume that none of the gold is lost in the process)

- A) $\sqrt[3]{50\pi}$ mm B) $\sqrt[3]{100\pi}$ mm C) $\sqrt[3]{50\pi^2}$ mm D) $\sqrt[3]{100\pi^2}$ mm E) NOTA

15) A survey of 100 Mu Alpha Theta students revealed the following data:

- 45 played curricular sports
- 34 played musical instruments
- 15 participated on other academic teams
- 20 played curricular sports and musical instruments ONLY
- 7 played curricular sports and participated on other academic teams ONLY
- 6 played musical instruments and participated on other academic teams ONLY
- 1 played curricular sports and musical instruments, and participated on other academic teams

How many students did not play curricular sports, musical instruments, or participate on other academic teams?

- A) 6 B) 32 C) 41 D) 60 E) NOTA

16) Andrew, Sandip, and Viraj are chasing a cat in Ybor City. Each of them has a $33.\bar{3}\%$ chance of catching it every time they spot it. If they take turns trying to catch the cat - Andrew first, then Sandip, then Viraj, and the cycle repeats indefinitely, what is the probability that Viraj will be the first to catch the cat?

- A) $\frac{2}{27}$ B) $\frac{4}{27}$ C) $\frac{4}{19}$ D) $\frac{1}{3}$ E) NOTA

17) The wave function for an electron within a certain length L is given by $\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$. What is the product of the amplitude and the period of this wave?

- A) $\frac{n\pi}{L} \sqrt{\frac{2}{L}}$ B) $\frac{\sqrt{2L}}{n\pi}$ C) $\frac{2\sqrt{2L}}{n}$ D) $\frac{4L}{n}$ E) NOTA

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- 18) A grain of sand, when at the point (x, y) of a coordinate plane, is allowed to move to either the point $(x + 1, y)$, or to the point $(x, y + 1)$. If the grain starts at $(0,0)$ and moves to $(4,4)$, find the number of possible distinct paths that could pass through $(2,2)$.
- A) 42 B) 30 C) 20 D) 12 E) NOTA
- 19) A sphere with a volume of 288π cubic inches is to be coated uniformly with lead. How thick must the lead be so that the surface of the exposed lead is exactly three times that of the original sphere?
- A) $6\sqrt{3} - 6$ inches B) 12 inches C) $18\sqrt{3} - 18$ inches D) 18 inches E) NOTA
- 20) A doorway is in the shape of a parabolic arch with equation $y = -2x^2 + 1$, with $y = 0$ meters representing the ground. What is the exact radius of the largest sphere that can be rolled through the doorway?
- A) $\frac{-3 + 3\sqrt{2}}{4}$ meters B) $\frac{2 + \sqrt{2}}{8}$ meters C) $\frac{-1 + 2\sqrt{2}}{4}$ meters D) $\frac{1}{2}$ meters E) NOTA
- 21) Aneeta's house is 6 miles west and 3 miles north of David Duchovny's mansion. Aneeta wants to go to the mansion, but she must first stop at a river to pick Beryl up on the way. If the river runs East-West and is 2 miles south of the mansion, how far, in miles, will Aneeta travel if she wants to get to David's with Beryl by the shortest route possible? (Assume that Aneeta will call Beryl and tell her where to meet along the river)
- A) $\sqrt{45}$ B) $\sqrt{85}$ C) $\sqrt{34} + \sqrt{13}$ D) $\sqrt{61} + \sqrt{4}$ E) NOTA
- 22) The St. Louis Arch has a base span of 522 feet. Assuming that the arch is in the shape of an upside-down parabola, and that it is 560 feet tall 70 feet horizontally from its vertex, how tall is the arch? (Round to the nearest tenth of a meter)
- A) 603.1 feet B) 603.2 feet C) 603.3 feet D) 603.4 feet E) NOTA
- 23) Jane and Maui are swimming in the ocean, 200 yards from the beach. When clouds start to approach, they decide to go get their towels and head in. They can both walk on the beach at 110 yards per minute but can only swim at 70 yards per minute. Their towels are on the edge of the beach next to the ocean, 100 yards from the perpendicular of the girls to the beach. To what distance on the beach from the towels should they swim to make landfall, in order to get to their towels in the minimum amount of time?
- A) 20 yards B) 40 yards C) 60 yards D) 80 yards E) NOTA

- 24) A spaceship is approaching Mars. It fires its retrorockets, causing it to slow down, stop, rise up again, then come back down. Its displacement, y kilometers, from the surface, is found to be: $y = -.01t^3 + .9t^2 - 25t + 250$, where t is time in seconds since the retrorocket was fired. The velocity at any time t can be expressed by $v(t) \approx \frac{y(t + \phi) - y(t - \phi)}{2\phi}$, where ϕ is any value. Find the hundredth's digit of the best approximation for the velocity when the spaceship touches the surface of Mars. (Note: $v(t)$ is approximated best as ϕ approaches 0)
- A) 3 B) 2 C) 1 D) 0 E) NOTA
- 25) A plane flies from New Jersey to San Diego at a speed of 300mph. It then returns to New Jersey at 350mph. Finally, it goes back to San Diego at 400mph. What was the average speed of the plane over the three segments to the nearest tenth mph?
- A) 345.2 B) 347.6 C) 348.2 D) 350.0 E) NOTA
26. Suppose you are working on a ranch and have 1000 yards of fencing material. You are told to build a circular fence around a mudhole of pigs and use the remainder of the fencing material to build a square corral. If you must use all of the fencing material, what will the length of the diagonal of the corral be if the total area of both is to be held to a minimum? (Round to the nearest yard.)
- a) 280 b) 198 c) 140 d) 100 e) NOTA
- 27) If a cylinder is inscribed in a sphere so that the cylinder has maximum volume, and the ratio $\frac{\text{Volume}_{\text{sphere}}}{\text{Volume}_{\text{cylinder}}}$ is expressed as $p\sqrt{q}$, where p and q are relatively prime integers, find $|p - q|$.
- A) 0 B) 1 C) 2 D) 3 E) NOTA
- 28) Alexandra is getting ready for prom, and she still needs to curl her hair and do her makeup. In her bathroom, there are 4 drawers to the left of the sink and 3 drawers on the right. She forgot which drawers she put her makeup and curling iron in, but she knows that she has makeup in 2 of the drawers on the left and 1 of the drawers on the right. She also has two curling irons, each in drawers on opposite sides of the sink. She then notices that all but 1 drawer on each side of the sink is locked! If a curling iron is not in the same drawer as makeup, what is the probability that Alexandra ends up being able to do her makeup and her hair?
- A) $\frac{1}{4}$ B) $\frac{1}{6}$ C) $\frac{1}{8}$ D) $\frac{1}{12}$ E) NOTA

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29) At Domino's Pizza, it costs \$13.24 for the first pizza and 49¢ less per pizza for the number of additional pizzas ordered. How many pizzas should you order if you want to pay the maximum amount possible?

- A) 15 B) 14 C) 13 D) 12 E) NOTA

30) Imagine that Raymond James Stadium is represented by the polar graph $r = \sqrt{\frac{16}{1 + 3\sin^2 \theta}}$. Thom can only fully grasp graphs in Cartesian form. If he successfully converted this polar graph to the Cartesian form: $ax^2 + bxy + cy^2 + dx + ey + f = 0$, find $acf + bd$.

- A) 64 B) 48 C) -48 D) -64 E) NOTA