2

1999 Mu Alpha Theta National Convention
Applications: Word Problems
Alpha Division

1) A group of students from Lakota West went out for a snack and had a total bill of $17.50 which they were to split equally. When El Cheepo and S. Kinflint left without paying, the remaining students had to pay $1.00 more. How large was the original party?

(A) 5 (B) 7 (C) 8 (D) 12 (E) NOTA

2. A very generous company wants to donate a sufficient sum of money to Mu Alpha Theta to provide a $1000 scholarship to be awarded every other year indefinitely. The scholarship is awarded the first year and the remaining money is to be deposited into an account earning 5% compounded annually. What is the minimum amount that the company can donate to Mu Alpha Theta to perpetuate the scholarship?

(A) $9,756.10 (B) $10,500.08 (C) $10,756.12 (D) $11,756.15 (E) NOTA

3. A bus company serves 500 customers and charges $0.50 for each bus ride. The company would like to increase the fare but they know that each increase of one cent reduces the number of riders by 10 people. What increase should they make to maximize their income.

(A) No increase (B)Inc. of 2 cents (C) Inc. of 4 cents (D) Inc. of 5 cents (E) NOTA

4. Four girls decided to use the same teeter - totter. The ones weighing 75 pounds and 50 pounds sat on opposite ends of the 12 - foot board which had a balance point at the center. The girl weighing 60 pounds got on the same side as the one weighing 50 pounds and sat 5 feet from the fulcrum. How far must the fourth girl sit from the fulcrum so that they balance if she weighs 40 pounds?

(A) $1\frac{3}{4}$ ft. (B) $2\frac{3}{4}$ ft. (C) $3\frac{3}{4}$ ft. (D) 5 ft. (E) NOTA

5. Bill, Bob, and Barry are hired to paint signs. In 8 hours Bill can paint 1 sign, Bob can paint 2 signs, and Barry can paint $1\frac{1}{3}$ signs. They all come to work the first day, but Barry doesn't like the job and quits after 3 hours. Bob works half an hour longer than Barry and quits. How long will it take Bill to finish the 2 signs they were supposed to paint?

(A) 1 hour (B) $1\frac{1}{2}$ hours (C) 2 hours (D) $2\frac{1}{2}$ hours (E) NOTA

6. A farmer wishes to fence a rectangular area behind his barn. The barn forms one short side of the rectangle and the length of the rectangle is 3 times the width. How many linear feet of fencing must he buy if the perimeter of the rectangle is 320 feet?

(A) 200ft. (B) 240ft. (C) 280ft. (D) 320ft. (E) NOTA

7. Mary drove 156 miles at a constant rate. If she had driven 9 mi / hr faster, she could have reduced the driving time by 45 minutes. Find the rate at which she actually drove.

(A) 38 mph (B) 39mph (C) 48mph (D) 49mph (E) NOTA

8. What is the smallest base in which 213 is odd and a perfect square?

(A) 3 (B) 4 (C) 6 (D) 8 (E) NOTA

9. Mary bought a math book and then sold it to Alice at a 20% loss. Alice sold the book to Joe and lost 20% on the deal. Joe made a 50% profit by selling the book to Grace for $5.28. How much did Mary pay for the math book?

(A) $5.40 (B) $5.60 (C) $5.80 (D) $5.90 (E) NOTA

10. A cathedral tower 100 feet high is 200 feet from a church tower 60 feet high. On the top of each tower is a pigeon. The two pigeons fly off at the same time and at the same speed directly to some grain on the level straight road that connects the two towers. The pigeons reach the grain at the same instant. How far is the grain from the foot of the cathedral tower?

(A) 81 ft (B) 82 ft (C) 83 ft (D) 88 ft (E) NOTA

11. If α is a first quadrant angle and $\cos\alpha = \frac{\sqrt{10}}{10}$, find $\sin 2\alpha$.

(A) $\frac{3\sqrt{10}}{5}$ (B) $\frac{3}{5}$ (C) $-\frac{4}{5}$ (D) $-\frac{3}{4}$ (E) NOTA

12. Find the area of the triangle with vertices $(3,4),(-2,5)$, and $(7,-1)$.

(A) 8.5 (B) 10.5 (C) 12.5 (D) 14.5 (E) NOTA

13. Melanie is standing on a tower and holding a ball 90 feet above the ground. She drops the ball and determines that after each bounce, it rebounds only two - thirds of its previous distance. How far has the ball traveled when it comes to rest?

(A) 270 feet (B) 360 feet (C) 450 feet (D) 540 feet (E) NOTA

14. A rectangular garden with dimensions u and v is bordered by a walk of uniform width w. The area of the walk is

(A) $4w^2$ (B) $2w(u+v)$ (C) $(v+2w)(u+2w)$ (D) $(u+w)(v+w)-uv$ (E) NOTA

15. An amoba divides into two amobas once every hour. How long, in hours, will it take for a single amoba to become a colony of 32,768 amobas?

(A) 8 hours (B) 12 hours (C) 16 hours (D) 17 hours (E) NOTA

16. One of the corner sections of a football stadium has one seat in the first row, three seats in the second row, five seats in the third row, and so on. If this section of the stadium will hold 1600 people when full with one person per seat, how many people can sit in the last row, one person per seat?

(A) 65 people (B) 79 people (C) 101 people (D) 159 people (E) NOTA

17. An urn is filled with coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

(A) 32 (B) 46 (C) 48 (D) 52 (E) NOTA

18. The electrical resistance of a wire varies directly as the length and inversely as the square of the diameter of the wire. If 65 m of wire of diameter 3 mm has a resistance of 10 ohms, what is the resistance, in ohms, of 50 m of the same type of wire if the diameter is 5 mm?

(A) $\frac{36}{13}$ (B) $\frac{60}{13}$ (C) 6 (D) $\frac{180}{13}$ (E) NOTA

19. If ABCD is a trapezoid with $\overline{DC}$ parallel to $\overline{AB}$, $\prec DCB$ is a right angle, DC = 6, BC = 4, AB = y, and the $m \prec ADB = x$. Find y in terms of x.

(A) $y = \frac{13 \sin x}{2\cos x + 3\sin x}$ (B) $y = \frac{26 \sin x}{2\cos x + 3\sin x}$

(C) $y = \frac{2\sqrt{13}(\sin x)}{4\cos x + 3\sin x}$ (D) $y = \frac{2\sqrt{13}(\sin x)}{3\cos x + 4\sin x}$ (E) NOTA

20. A circle is inscribed in a square, which is inscribed in a circle, which is inscribed in an equilateral triangle, which is inscribed in another circle. If the area of the smallest circle is π, what is the area of the largest circle?

(A) 4π (B) $4\sqrt{3}\pi$ (C) 8π (D) $6\sqrt{2}\pi$ (E) NOTA

21. The arithmetic mean between two positive numbers is 17, and their positive geometric mean is 15. Find the positive difference between the two numbers.

(A) 10 (B) 12 (C) 14 (D) 16 (E) NOTA

22. If $f(x) = 2x^2 - 3$ and $g(x) = \sqrt{x+1}$, find $[g \circ f](x)$.

(A) $2x^2 - 2$ (B) $2x - 1$ (C) $\sqrt{2x-1}$ (D) $\sqrt{2x^2 - 2}$ (E) NOTA

23. An 8" X 8" X 8" box is tightly packed with 16 identical cylinders which are 8" high and have a diameter of 2". How many cubic inches of space remain in the box?

(A) $16(4 - \pi)$ (B) $64(\pi - 1)$ (C) $128(4 - \pi)$ (D) $512(\pi - 1)$ (E) NOTA

24. On 8 / 2 / 99 at 9 AM, Joe noted that 1000 bacteria in a certain culture A were present. At 1 PM, on the same day, Joe stated that the same culture A had 3000 bacteria present. Assuming exponential growth, at what time would the bacteria in this culture be 243,000?

(A) 5AM on 8 / 3 / 99
(B) 9 AM on 8 / 3 / 99
(C) 1 PM on 8 / 3 / 99
(D) 5 PM on 8 / 3 / 99
(E) NOTA

25. A survey of students' favorite sports revealed the following:

71 liked baseball
43 liked football
47 liked basketball
24 liked basketball and football
33 liked baseball and football
38 liked baseball and basketball
20 liked all three

How many students were surveyed if we assume that all of the students liked something?

(A) 55 (B) 86 (C) 104 (D) 276 (E) NOTA

26. The Pederson's garden has a triangular area enclosed by three sidewalks. The triangle has sides of 4', 5' and 7'. Mrs. Pederson wants to install a circular pool of the largest possible area in the triangular region. What will the area of the pool be?

(A) $\frac{\pi\sqrt{6}}{2}$ (B) $\frac{3\pi}{2}$ (C) $4\sqrt{6}$ (D) $\frac{3\pi}{8}$ (E) NOTA

27. George Washington was born 11 years before Thomas Jefferson. In 1770 Washington's age was 3 years more than 7 times the age of Jefferson in 1748. What was the sum of the two men's ages in 1750?

(A) 25 (B) 26 (C) 27 (D) 28 (E) NOTA

28. An ant is on the edge of the top of a cylinder. The ant wants to reach a point diagonally across from him and on the bottom of the cylinder. If the cylinder is 8" tall and has a diameter of 4", what is the least number of inches the ant must crawl in order to reach his destination?

(A) $4\sqrt{5}$ (B) $2\sqrt{(16+\pi^2)}$ (C) $4\sqrt{(4+\pi^2)}$ (D) $8+2\pi$ (E) NOTA

29. The tens digit of a two - digit number is five more than the units digit. If 3 is subtracted from the number and 2 is added to the reversed number, the former will be twice the latter. What is the number?

(A) 50 (B) 61 (C) 72 (D) 83 (E) NOTA

30. Hank and Anna open a print shop. They receive a job which Hank can do in 12 hours or Anna in 14 hours. They start to work on it together, but after 3 hours, Anna has to stop to finish another job. Hank works alone for an hour, when he is called out for an extimate. Anna comes back and finishes the job alone. How long will it take her to finish?

(A) $6\frac{1}{3}$ hrs. (B) $6\frac{2}{3}$ hrs. (C) $7\frac{1}{3}$ (D) $7\frac{2}{3}$ (E) NOTA

T.1 **Mary, Sue, and Bill work at a motel. It would take Mary 10 hours (if she worked alone), Sue 8 hours, and Bill 12 hours to clean the whole motel. One day Mary came to work early and she had cleaned for 2 hours when Sue and Bill arrived and all three finished the job. How long did they take to finish?**

T.2 **A farmer wishes to fence a rectangular area behind his barn. The barn forms one end of the rectangle and the length of the rectangle is three times the width. How many linear feet of fence must he buy if the perimeter of the rectangle is 320 feet?**

T.3 **The tens digit is two less than the units digit. If the digits are reversed, the sum of the reversed number and the original number is 154. Find the original number.**