

## Precalculus Team Vero Beach Invitational January 24, 2004

1. In a survey of 75 students, the following results were given: 22 took precalculus; 33 took statistics but not precalculus, 19 took only trig; 15 took precalculus and trig; 24 took trig and statistics; 9 took trig and precalculus, but not statistics; and 6 took statistics and precalculus, but not trig.

A = the number of students who did not take a math class

B = the number of students who took only one math class.

Find the value of  $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

2. Write the equation of the line in standard form ( $Ax + By = C$ , where A, B, and C are relatively prime) that is perpendicular to the line  $6x - 3y = 7$  and the line contains the intersection of  $y - 3x = 2$  and  $2x + 5y = -7$ .

3.  $f(x) = 4\cos\left(3x - \frac{2\pi}{3}\right) - 6$

A = the phase shift

B = the period

C = Amplitude

D = the maximum value

Find  $\cos^{-1}\left(\left(\frac{\sqrt{B}}{\sqrt{A}}\right)\frac{D}{C}\right)$

4. A car traveled 3 miles from point A to point B on a bearing of N78°E. It then traveled 4 miles to point C on a bearing of S42°E. Find the (exact) straight-line distance from point C to point A.

5. Find the exact distance between the polar points  $\left(5, \frac{11\pi}{12}\right)$  and  $\left(9, \frac{7\pi}{12}\right)$ .

6. A = |slope of asymptotes of graph of  $4x^2 - y^2 - 48x - 4y + 124 = 0$ ]

B = eccentricity of graph of  $4x^2 - y^2 - 48x - 4y + 124 = 0$

C = length of the latus rectum of  $8y = -x^2 - 6x + 5$

D = the ordinate of the focus of the graph of  $8y = -x^2 - 6x + 7$

Find AC - BD

7. As Dave sat writing at the kitchen table yesterday, he noticed a ladybug, Dotty, crawling at a steady pace across a piece of graph paper in a remarkably straight line. She seemed to be heading straight for a gentlemanbug behind Dave's half-eaten sandwich. Dave immediately grabbed his stopwatch and timed her. Dotty progressed from (2, -1) to (5.2, 0.6) in 2 seconds and was at (10, 3) exactly 3 seconds later (5 seconds after time started). At this rate, how long, in seconds, will it take Dotty to travel from (2, -1) to (30, 13)?

8.

$$A = \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i$$

$$B = \sum_{i=12}^{85} 4i - 8$$

$$C = \sum_{i=0}^{10} 2^i$$

$$D = \sum_{i=6}^{25} i^2$$

Find  $\frac{B}{A} + D - C$

Precalculus Team Vero Beach Invitational January 24, 2004 page 2

9. Given  $f(x) = 4x^5 - 12x^4 - 17x^3 + 63x^2 + 4x - 60$

A = Sum of roots of  $f(x+1)$

B = Product of roots of  $f(x-2)$

C = Sum of the reciprocals of the roots of  $f(x)$

Find  $AB - C$

10. A = the probability of tossing at least 4 heads when tossing 6 fair pennies

B = the probability that Mrs. Hiller, Mr. Koski and Dr. Morris will all be sitting together if a group of eight math team sponsors (including these three) are seated randomly in a row.

C = the probability of choosing at least 1 green sock when selecting 3 socks from a drawer containing 3 green, 3 red, and 2 blue socks

D = the probability of rolling a product of 20 or higher when rolling a pair of fair dice given that at least one of the die is a six.

Find  $\frac{ABD}{C}$

11. Given:  $\log 2 = m$ ;  $\log 3 = n$ ; and  $\log 7 = p$

A =  $\log 30$

B =  $\log 147$

C =  $\log 10.5$

D =  $\log 2160$

In terms of  $m$ ,  $n$ , and/or  $p$ , find the value of  $A + B + C - D$

12. Let A = the sum of the coefficients of the expansion on  $(2x - 4y)^5$

Let B = the coefficient of the 6th term of the expansion of  $(x + 2y)^9$

Let C = the constant term of the expansion of  $\left(x^3 + \left(\frac{2}{x}\right)^2\right)^5$

Find  $\frac{B}{A} + C$

13. Find the exact values of the two square roots of  $-2 + 2\sqrt{3}i$  in  $a + bi$  form.

14. Bill, Bob, and Bubba are trying to see who can make the first basketball shot from mid-court. The odds of Bob making a shot are 1:8, Bill's odds of making a shot are 1:6, and Bubba's odds are 1:4. If the players shoot in the order of Bill, then Bob, then Bubba until someone makes a shot, what the probability that Bubba will make the first shot?

15. The graph of function  $f(x)$  is a line segment joining the two points  $(-3,1)$  and  $(5,6)$ . Determine the slope of the line segment that results from graphing the equation of  $y = -0.5f^{-1}(x)$ . (Note: The notation  $f^{-1}$  is the inverse of  $f(x)$ ).