

Geometry Individual Solutions  
Vero Beach March 19, 2005

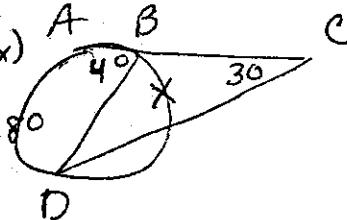
1. B  $\Delta$  is isosceles right,  
Legs are 4,  $A = \frac{1}{2} \cdot 4 \cdot 4 = 8$

2. B  $\frac{60h - 11m}{2} < 90$   
 $m > 32.7$

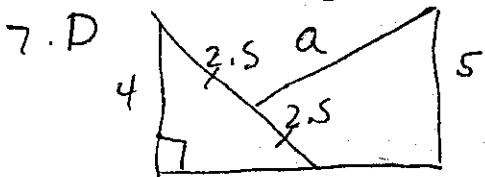
3. A  $\sqrt{\frac{(3-4)^2 + (1-6)^2 + (0-2)^2}{1+25+4}}$   
 $\sqrt{130}$

4. D  $\Delta$  are  $\sim$  by SSS  
Ratio of sides  $\frac{1}{2}$   
Ratio of areas  $\frac{1}{4}$   
so (4) will fit

5. D  $m\angle C = \frac{1}{2}(m\widehat{AC} - x)$   
 $m\widehat{AD} = 80$   
 $30 = \frac{1}{2}(80 - x)$   
 $x = 20$   
 $m\angle D = 10$



6. E. Let orig  $r = 10$ .  $V = \frac{4000}{3}\pi$ . when  
 $r$  decreased by 20%, new radius = 8.  
 $V = \frac{2048}{3}\pi$ , decrease is  $\frac{1952}{3}\pi$ .  
 $\frac{\frac{1952}{3}\pi}{\frac{4000}{3}\pi} = 48.8\%$

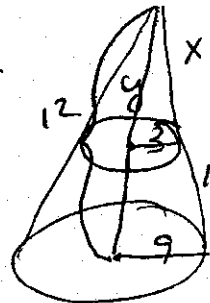


7. D  
 $P = 16.5 + a$   
 $K = \frac{16.5 + a}{2}$

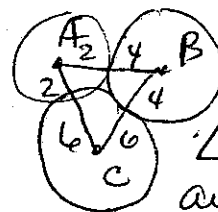
$C = 2(16.5 + a)\pi$   
 $= (16.5 + a)\pi$

8. D  $\frac{AC \cdot DB}{AC + DB} = \frac{30 \cdot 20}{50} = 12$

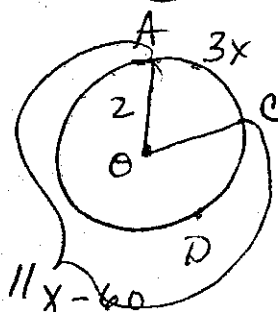
9. C  $\frac{1}{3} = \frac{x}{x+10}$   
 $x = 5$   
Use Pythag.  
find ht is 12  
 $\frac{y}{12} = \frac{1}{3}$ ,  $y = 4$   
ht of frustrum is  $12 - 4 = 8$



10. D Sides of  $\Delta$   
are 6, 8, 10.  
 $\Delta$  is right  
area =  $\frac{1}{2} \cdot 6 \cdot 8 = 24$



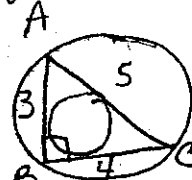
11. C



$11x - 60$   
 $11x - 60 + 3x = 360$   
 $x = 30$

$m\widehat{AC} = 90$ ,  $m\angle AOC = 90$   
area of sector =  $\frac{1}{4} \cdot 4\pi = \pi$

12. D



$AC = 5$  is diameter of  $\odot$ .  
since  $m\angle B = 90$ .  
 $r = \frac{5}{2}$ , radius of inscribed  $\odot$  is Area of  $\Delta$   
 $\frac{\text{Area of } \Delta}{\text{semiperim } \Delta} = \frac{1}{2} \cdot \frac{3 \cdot 4}{6} = 1$

$\frac{5}{2} - 1 = \frac{3}{2}$

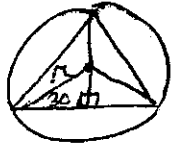
13. C Find V of outer  $\odot$ , subtract inner  $\odot$ , then subtract V of 3 holes.

$$\frac{1}{3}(81\pi - 36\pi) - 3\left(\frac{1}{3}\pi \cdot \frac{1}{16}\right) = \frac{239}{16}\pi$$

Set = to V of sphere to find  $r$ .  $\frac{239}{16}\pi = \frac{4}{3}\pi r^3$

$$r = \frac{\sqrt[3]{717}}{4}$$

14. B



Circumference  $C$   
has a radius of  $\frac{C}{2\pi}$ .  
In the  $\Delta$ , side opp  $60^\circ$   
is  $\frac{1}{2} \cdot \frac{C}{2\pi} \cdot \sqrt{3} = \frac{C\sqrt{3}}{4\pi}$

$\frac{C\sqrt{3}}{4\pi} \cdot 2 = \text{side of } \Delta$

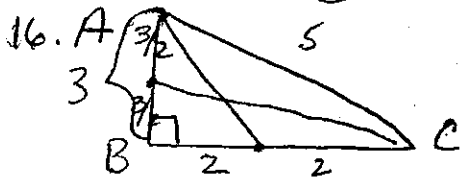
$$\frac{4\pi}{C\sqrt{3}} \cdot 3 = \text{perimeter}$$

15. C  $a = \frac{kc^2}{b}$ ,  $k = \frac{ab}{c^2}$

$$\frac{10 \cdot 5}{25} = \frac{a \cdot 12}{64}, a = \frac{32}{3}$$

$$P = 20 + \frac{32}{3} = \frac{92}{3}$$

$$S = \frac{92}{3} \cdot \frac{1}{2} = \frac{46}{3}$$



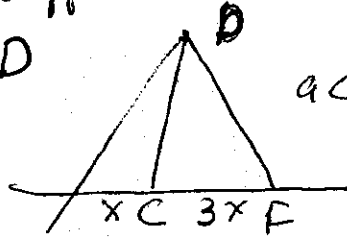
med to hyps =  $\frac{5}{2}$

$$\text{med to BC} = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\text{med to AB} = \sqrt{\left(\frac{3}{2}\right)^2 + 4^2} = \frac{\sqrt{37}}{2}$$

$$\left(\frac{5}{2}\right)^2 + (\sqrt{13})^2 + \left(\frac{\sqrt{37}}{2}\right)^2 = 37.5$$

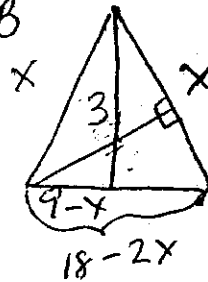
17. E In a quadrilateral inscribed in a  $\odot$ , opposite  $\angle$ s are supplementary, so difference is  $\odot$ .



$$A_{\Delta DEF} = 6$$

Since alts are the same, ratio of areas is the ratio of the bases. Bases are  $\frac{1}{2} = \frac{x}{6}$ .  
Area DEF = 2.  $A_{\Delta DEC} = 2 + 6 = 8$ .

19. B



$$p = 18$$

$$9 + (9-x)^2 = x^2$$

$$x = 5$$

$$9 - x = 4$$

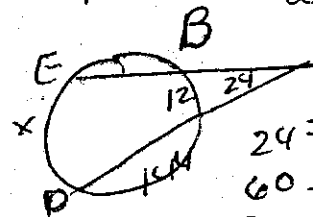
$$A = \frac{1}{2} \cdot 3 \cdot 8 = 12$$

20. C  $A = 11 - 2 = 9$

$$A = \frac{1}{2} \cdot 6 \cdot 9 = 27$$

21. B Area of  $\Delta = 36\sqrt{3}$ .  $36\sqrt{3} = 6 \cdot \frac{a^2\sqrt{3}}{4}$   
makes side of hex  $2\sqrt{6}$ . Apothem would be alt of  $\Delta$  formed by drawing diag. Apothem opp  $60^\circ \angle$ ,  $3\sqrt{2}$ .

22. C

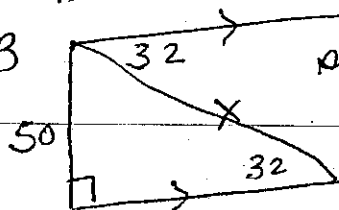


$$24 = \frac{1}{2}(x - 12)$$

$$60 = x$$

$$BE = 360 - (60 + 144 + 12)$$

23. B



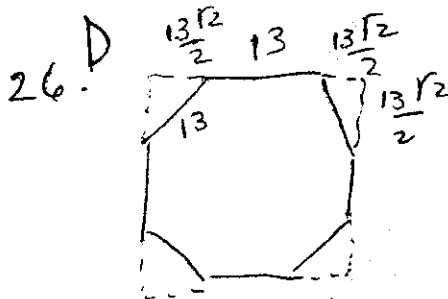
$$\sin 32 = \frac{50}{x}$$

$$x = 94.35$$

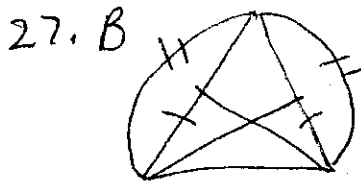
24. B Platonic Solids

hexahedron, octahedron  
tetrahedra, dodecahedron  
icosahedron

25. C Since diagonals in a rhombus are  $\perp$ , use Pythag to find other diag = 40.  $A = \frac{1}{2}d_1d_2 = 600$ . Alt of rhombus is diameter of circle. Use area backwards,  $600 = 25 \cdot h$ ,  $A = 24 \cdot A_{\square} = 144\pi$ , area bounded region is  $600 - 144\pi$ .

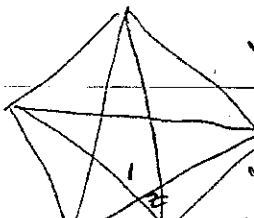


Area of  $\square = a^2$   
 $(13 + 13\sqrt{2})^2 - 4 \cdot \frac{1}{2} \cdot 13\sqrt{2} \cdot 13\sqrt{2}$   
 $169 + 338\sqrt{2} + 338 - 169$   
 $338 + 338\sqrt{2}$



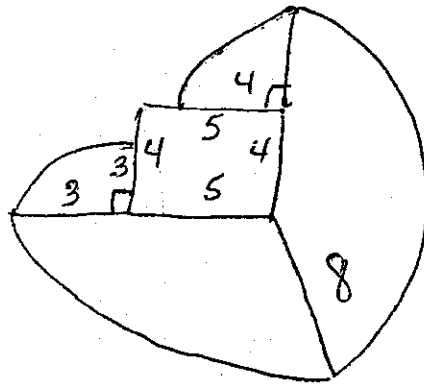
$\angle ACB \cong \angle ABC$   $\triangle, \triangle$   
 $BC = CB$  reflexive  
 $\triangle CDB \cong \triangle BEC$  SAS  
 $DF = EC$  def of  $\cong \triangle$   
 or  
 CPCTC

28. C



Draw all diagonals  
 Inner pentagon has interior  $\angle$  of 108.  
 $\angle MLE \cong 72$ .

29. B



$\frac{3}{4} \cdot 64\pi + \frac{1}{4} \cdot 9\pi + \frac{1}{4} \cdot 16\pi$   
 $\frac{217\pi}{4}$

36. B

$\frac{1}{3} Bh = \frac{4}{3} \pi r^3$   
 $\frac{1}{3} \pi r^2 h = \frac{4}{3} \pi r^3$   
 $A = \frac{4}{3} \pi r^3$   
 $\frac{1}{3} \pi r^2$   
 $= 4r$