# Vero Beach Invitational March 25, 1995
## Geometry Solutions

C 1. P of △ is 20, P of △ formed by joining midpoints is $\frac{1}{2}P = 10$. Segment formed by midpts is ∥ to & $\frac{1}{2}$ length of 3rd side.

B 2. r of ⊙ = 4 & this is diameter of O′, radius of O′ = 2, area of ⊙′ = $4\pi$

C 3. $180 - 83 = 97$

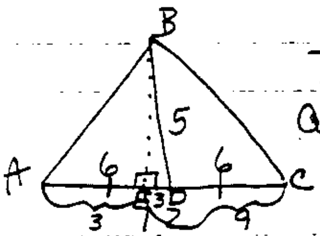
C 4. Slopes of ∥ lines are the same. $\frac{5}{2} = 2.5$

D 5. $2x + 3x + 5x = 180$

$x = 18$

Smallest interior is 36. Therefore largest exterior is $180 - 36 = 144$

A 6.




Draw alt.

Area = $\frac{1}{2}bh$

$24 = \frac{1}{2} \cdot 12 \cdot h$

$4 = h$

BE = 4. Use Pythag. Thm. to find AB & BC.

$AB^2 = 3^2 + 4^2$ &, $AB = 5$

$BC^2 = 9^2 + 4^2$, $BC = \sqrt{97}$

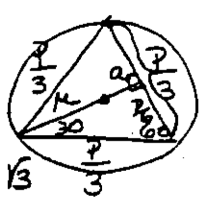
A 7. $9\pi = \frac{1}{3}Bh$ $r = h$

$9\pi = \frac{1}{3}\pi r^2 \cdot r$

$27 = r^3$

$3 = r$

~~D~~ C 8. side = $\frac{P}{3}$, center of ⊙ is pt where medians ∩, radius of ⊙ is radius of △, $a + r = \frac{P}{6}\sqrt{3}$



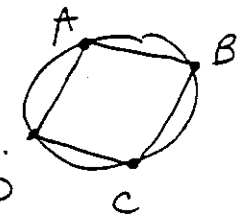

$r = 2a$ (3 equal parts) which makes $r = \frac{1}{9}P\sqrt{3}$

$A = \pi r^2$

$A = \pi\left(\frac{1}{9}P\sqrt{3}\right)^2$

$= \frac{\pi P^2}{81}$

D 9. opp ∠s are supplementary.




$m\angle A + m\angle C = 180$

$7x + 20 + 3x + 40 = 180$

$10x = 120$

$x = 12$

$m\angle B = 125$

E 10. $\sqrt{(-2-4)^2 + (3 - -6)^2}$

$\sqrt{36 + 81}$

$\sqrt{117}$

$3\sqrt{13}$

D 11. ∠1 & ∠2 are same side interior angles & are supplementary

$4x - 2 + 2x + 2 = 180$

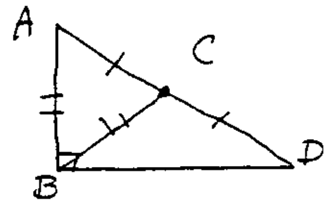
$6x = 180$

$x = 30$

Since ∠1 ≅ ∠3 (alt int ∠s)

$m\angle 3 = 118$

B 12. Since △ABD is right, C, the midpt of hypotenuse is equidistant from all vertices. Making AB=BC=AC, & △ABC is equilateral.




$m\angle DAB = 60$

# Geometry Individual Solutions - March 25, 1995



B 13. $\frac{1}{2}bh = h(\text{median})$

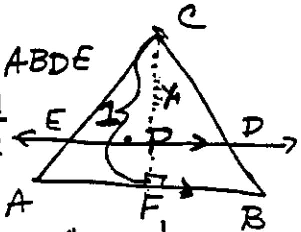
heights are =

$\frac{1}{2}b = .(\text{median})$

$\frac{1}{2} \cdot 18 = .(\text{median})$

$9 = \text{median}$

B. 14. $a\triangle CDE = a\square ABDE$




So $\frac{a\triangle CDE}{a\triangle ABC} = \frac{1}{2}$

ratio of sides of $\triangle$'s $= \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{x}{1}$, $x\sqrt{2} = 1$, $x = \frac{\sqrt{2}}{2}$

distance from P to $\overline{AB}$ is $1-x$ or $1-\frac{\sqrt{2}}{2}$, this equivalent to $\frac{2-\sqrt{2}}{2}$.

E 15. $x^2+6x+y^2-2y=10$

$x^2+6x+9+y^2-2y+1 = 10+9+1$

$(x+3)^2+(y-1)^2 = 20$

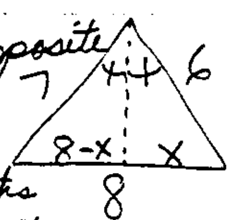
center $(-3,1)$ $r=\sqrt{20}$ or $2\sqrt{5}$

area of circle is $20\pi$

B 16. From 6 a.m. to 3 p.m. hr hand moves from the 6 to the 3, which is $\frac{3}{4}$ of the circumference of clock.

$\frac{3}{4} \cdot 24\pi = 18\pi$ in

D 17. Largest $\angle$ is opposite longest side, 8.




$\angle B$ divides side into lengths proportion to sides.

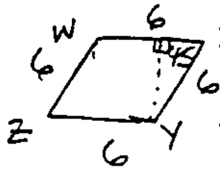
$\frac{6}{x} = \frac{7}{8-x}$

$48-6x=7x$ $\qquad 8-\frac{48}{13} = \frac{56}{13}$

$\frac{48}{13} = x$

so $\frac{48}{13}$ is smallest segment.

C 18.



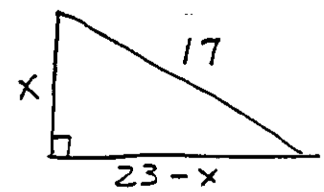

Draw altitude to $\overline{WX}$.

Its measure is $3\sqrt{2}$.

area $= 6 \cdot 3\sqrt{2}$

$= 18\sqrt{2}$

B 19. hyp = $40-23=17$




Using Pythag.

$x^2+(23-x)^2 = 17^2$

$x^2+529-46x+x^2=289$

$2x^2-46x+240=0$

$2(x^2-23x+120)=0$

$2(x-8)(x-15)=0$

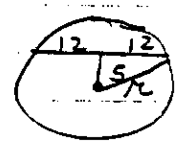
$x=8, 15$

area $= \frac{1}{2} \cdot 8 \cdot 15$

$= 60$

C 20. $r=13$

$C = 26\pi$




D 21. $230 = \frac{n(n-3)}{2}$

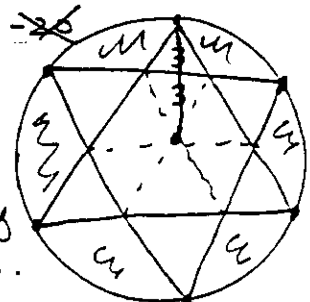
$460 = n^2-3n$

$0 = n^2-3n-460$

$0 = (n-23)(n+20)$

$n = 23, \cancel{-20}$

D 22. This is one solution.




Area of shaded = area of $\odot$ − area 12 ≅ equi $\triangle$.

Radius of $\odot$ is 6, shown segments would = Since alt of 1 $\triangle$, side of $\triangle = 2\sqrt{3}$.

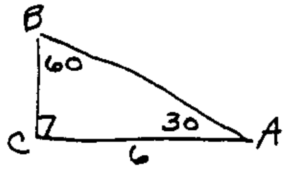
$a\odot = 36\pi$

$a\triangle = 12 \cdot \frac{4 \cdot 3\sqrt{3}}{4}$

$36\pi - 36\sqrt{3}$ or $36(\pi-\sqrt{3})$

# Geometry Individual Solutions – March 25, 1995

**B 23.**



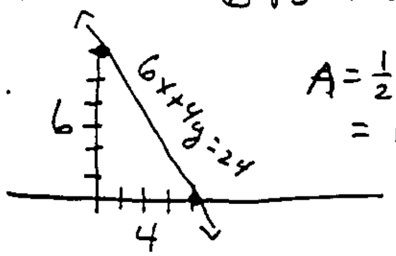

$BC = \frac{6}{\sqrt{3}}$ which is $2\sqrt{3}$

$AB = 2 \cdot 2\sqrt{3}$ which is $4\sqrt{3}$

$p = 4\sqrt{3} + 2\sqrt{3} + 6$

$= 6\sqrt{3} + 6$

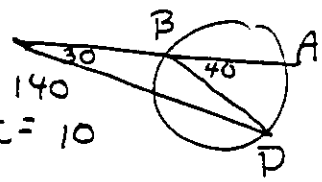
**B 24.**




$A = \frac{1}{2} \cdot 4 \cdot 6$

$= 12$

**D 25.**




$m\angle CBD = 140$

$m\angle BDC = 10$

**B 26.** $-3(x + 3y = 7)$ Solve the system

$3x - y = 1$

$-3x - 9y = -21$

$-10y = -20$

$y = 2,\ x = 1 \qquad (1, 2)$

**C 27.** $d = \sqrt{l^2 + w^2 + h^2}$

$= \sqrt{25 + 49 + 16}$

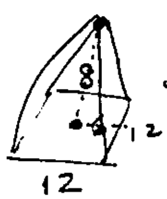
$= \sqrt{90}$ or $3\sqrt{10}$

**E 28.** $TSA = LA + B$

$TSA = \frac{1}{2}P + B$

$= \frac{1}{2} 48. + 144$

$= 384$




$l = 10$

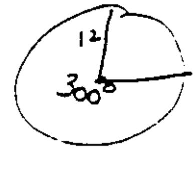
**A 29.** $140 = \frac{(n-2)180}{n}$

$140n = 180n - 360$

$-40n = -360$

$n = 9$

**B 30.**



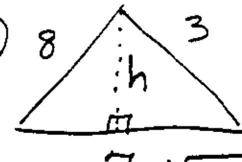

$\frac{300}{360} \cdot 144\pi$

$\frac{5}{6} \cdot 144\pi$

$120\pi$

## Team Solutions

$\frac{12\sqrt{3}}{7}$ **1.**




Find area using Heron's.

$\sqrt{s(s-a)(s-b)(s-c)}$

$\sqrt{9 \cdot 1 \cdot 2 \cdot 6}$

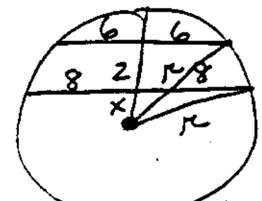
$6\sqrt{3}$

$6\sqrt{3} = \frac{1}{2} \cdot 7 \cdot h$

$\frac{2}{7} \cdot 6\sqrt{3} = h$

$h = \frac{12\sqrt{3}}{7}$

$100\pi$ **2.**




Using Pythag

$8^2 + x^2 = r^2$ and

$6^2 + (2+x)^2 = r^2$.

Set these = & solve.

$64 + \cancel{x^2} = 36 + 4 + 4x + \cancel{x^2}$

$24 = 4x$

$6 = x$

Find radius. $6^2 + 8^2 = r^2$

$r^2 = 100$

$A = 100\pi$

$\frac{5}{3}$ **3.** Find points of ∩.

$\begin{cases} y = 0 \\ x = 3y \end{cases} \quad (0, 0)$

$\begin{cases} y = 0 \\ 3x - y = 10 \end{cases} \quad (3\frac{1}{3}, 0)$

$\begin{cases} x = 3y \\ 3x - y = 10 \end{cases} \quad (3, 1)$

Then graph. (see next page)