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1. $A = f'(x)g(x) + g'(x)f(x) = (-3)(1) + (2)(3) = 3;\quad B = \dfrac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} = \dfrac{(-3)(1)-(2)(3)}{1} = -9$

$C = f'(g(x))g'(x) = f'(2)(3) = (-1)(3) = -3 \qquad D = g'(3)(-2) = 3(-2) = -6$

$\dfrac{AB}{CD} = \dfrac{3(-9)}{(-3)(-6)} = -\dfrac{3}{2} = -1.5$

2. Find the boundaries: $x^2 - 5x - 2 = 6 - 3x$.; $0\ x^2 - 2x - 8$; $x = -2, 4$

$A = \int_{-2}^{4}\left(6-3x-(x^2-5x-2)\right)^2 dx = \int_{-2}^{4}(8+2x-x^2)^2 dx = \dfrac{1296}{5};\quad B = \dfrac{\sqrt{3}}{4}\int_{-2}^{4}(8+2x-x^2)^2 dx = \dfrac{\sqrt{3}}{4}(\dfrac{1296}{5}) = \dfrac{324\sqrt{3}}{5}$

$C = \dfrac{1}{2}\pi\int_{-2}^{4}(.5(8+2x-x^2))^2 dx = \dfrac{162}{5}\pi; \qquad \dfrac{AB}{C} = \dfrac{1296}{5} \bullet \dfrac{324\sqrt{3}}{5} \bullet \dfrac{5}{162\pi} = \dfrac{2592\sqrt{3}}{5\pi}$

3. $A = \lim_{n\to\infty}\dfrac{2}{n}\sum_{i=1}^{n}\left(2+\left(\dfrac{2i}{n}\right)\right)^5 = \int_2^4 2x^5 dx = \dfrac{4032}{5} \qquad B = \lim_{x\to\frac{4\pi}{3}} \dfrac{\int_{\frac{4\pi}{3}}^{x}\cos t\,dt}{3x-4\pi} \Rightarrow \lim_{x\to\frac{4\pi}{3}}\dfrac{\cos x}{3} = \dfrac{-.5}{3} = -\dfrac{1}{6}$

$C = \lim_{x\to\frac{\pi}{3}}\dfrac{\sin(x)-0.5\sqrt{3}}{x-\dfrac{\pi}{3}} = \dfrac{d(\sin x)}{dx};\cos\left(\dfrac{\pi}{3}\right); C = .5 \qquad D = \lim_{x\to 0}\dfrac{3\sin 2x\cos 2x}{x} = \lim_{x\to 0}\dfrac{3\sin 4x}{2x} \Rightarrow \lim_{x\to 0}\dfrac{12\cos 4x}{2} = 6$

$C - A\int_{\frac{1}{B}}^{D}(\tan(3x) - \sin^3(2x) + \csc^5(7x))dx = .5 - 0 = .5$

4. $A = \sum_{n=2}^{\infty}\left(\dfrac{2}{3}\right)^n = \dfrac{\left(\dfrac{2}{3}\right)^2}{1-\dfrac{2}{3}} = \dfrac{4}{3}$

$B = \sum_{n=1}^{\infty}\dfrac{8}{4n^2-1} \qquad \dfrac{8}{4n^2-1} = \dfrac{A}{2n-1} + \dfrac{B}{2n+1} \quad A(2n+1) + b(2n-1) = 12;\ \ A = 4; B = -4$

$B = \sum_{n=1}^{\infty}\dfrac{8}{4n^2-1} = \sum_{n=1}^{\infty}\left(\dfrac{4}{2x-1} - \dfrac{4}{2x+1}\right) = \dfrac{4}{2(1)-1} - \dfrac{4}{2(1)+1} + \dfrac{4}{2(2)-1} - \dfrac{4}{2(2)+1} + \ldots = 4$

$C =$ the radius of convergence of $\dfrac{2}{2}x + \dfrac{4x^2}{4} + \dfrac{6x^3}{8} + \dfrac{8x^4}{16} + \ldots \qquad nth\ term = \dfrac{2nx^n}{2^n}$

By ratio test $\lim_{n\to\infty}\left(\dfrac{2n+2}{2^{n+1}}\dfrac{2^n}{2n}\right)\left|\dfrac{x^{n+1}}{x^n}\right| < 1$

$\dfrac{1}{2}|x| < 1;$ therefore radius $= 2$

Find: $A\int_C^B \dfrac{xdx}{x^2+8} = \dfrac{4}{3}\int_2^4\dfrac{xdx}{x^2+8} = \dfrac{1}{2}\left(\dfrac{4}{3}\right)\ln(x^2+8)\big|_2^4 = \dfrac{2}{3}(\ln 24 - \ln 12) = \dfrac{2}{3}\ln 2$

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5. $f'(x) = 4x^3-12x^2-16x$ $f''(x)=12x^2 - 24x-16$

x		-1	$\frac{3-\sqrt{21}}{3}$	0	$\frac{3+\sqrt{21}}{3}$	3	4	
f ′ (x)	-	0	+	0	-	-	0	+
f ″(x)	+	+	0	-	0	+	+	+

f′(x) < 0 and f″(x) > 0 on the intervals $(-\infty, -1) \cup (\frac{3+\sqrt{21}}{3}, 4)$

6. $A = \frac{8.0-0}{3(8)}[2.5+4(2.25)+2(1.94)+4(1.71)+2(1.52)+4(1.22)+2(1.02)+4(0.52)+.24] = \frac{23}{2}$

$B = \frac{8}{8}[2.44+2.01+1.83+1.55+1.45+1.18+0.86+0.29] = 11.61$

$C = \frac{8}{2(16)}(2.5+2(2.44+2.25+2.01+1.94+1.83+1.71+1.55+1.52+1.45+$

$1.22+1.18+1.02+0.86+0.52+0.29)+0.24) = 11.58$

$$\frac{A+B+C}{3} = 11.56\overline{3} = \frac{3469}{300}$$

7. $A(v) = \int_0^v (24x^2 - 12x^3)dx = 8v^3 - 3v^4$

$A = 1$; $5 = 8v^3 - 3v^4$ B =1.215 = ; $A(v) = 8v^3 - 3v^4; \frac{dA}{dt} = (24v^2 - 12v^3)\frac{dv}{dt}; \frac{dA}{dt} = (24\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right)^3)(.09)$

$C = \frac{4}{3}; A'(v) = 24v^2 - 12v^3; A''(v) = 48v - 36v^2; v = 0, \frac{4}{3};$ $ABC = 1 \bullet 1.215 \bullet \frac{4}{3} = 1.62$

8. $V = \pi r^3; \frac{dV}{dt}$

9..
$A = \frac{1}{7-2}\int_2^7 \left(30 - 30\sin\left(\frac{\pi t}{12}\right)\right)dx = 4.220;$ $B = \int_1^{1.75} \left(30 - 30\sin\left(\frac{\pi t}{12}\right)\right)dx = 14.587;$ $|4.220 - 14.587| = 10.367$

10. Area of one petal of the tabletop $= 0.5\int_0^{\frac{\pi}{3}} (6\sin(3\theta))^2 d\theta = 3\pi;$ 3 petals gives area $= 3(3\pi) = 9\pi$

Area of skirt $= \frac{2}{3} \bullet 3\int_0^{\frac{\pi}{3}} \sqrt{(6\sin(3\theta)^2 + (18\cos(3\theta)^2} d\theta \approx 376.991118$ Total rounded area = 405.265

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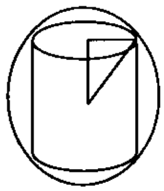
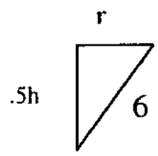
11. 



$V = \pi r^2 h;\ r^2 = 36 - \frac{1}{4}h^2;\ V(h) = \pi(36 - \frac{1}{4}h^2)h = \pi\left(36h - \frac{1}{4}h^3\right)$

$V' = \pi(36 - \frac{3}{4}h^2);\quad 36\left(\frac{4}{3}\right) = h^2; h = 12\sqrt{3};\ r^2 = 36 - 12 = 24$

$V = 24 \bullet 12\sqrt{3}\tau = 288\sqrt{3}\pi$

12. A= $160\pi^2$ $\quad 4(x-5)^2 + 16(y+4)^2 = 64$ Area of ellipse = $4(2)\pi$; C = $2(4+6)\pi = 20\pi$; AC=$160\pi^2$
B=$288\pi^2$ $\quad 9(x+5)^2 + 36(y+3)^2 = 1296$ $\quad$ A=$12(6)\pi$ C=$2(7-5)\pi$; V = AC=288π
$|160-288|\pi^2 = 128\pi^2$

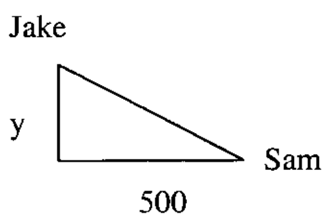
13. θ = the initial value + radians per minute the hr hand moves - radians per minute the minute hand moves. $\theta = \pi + \frac{\pi t}{360} - \frac{\pi t}{30}$. Area of a sector = $0.5\theta r^2$.

$A = \frac{9}{2}\left(\pi + \frac{\pi t}{360} - \frac{\pi t}{30}\right)$; Average value $= \frac{1}{30}\int_0^{30}\left(\pi - \frac{11\pi t}{360}\right)dt = 7.658$

B= 6:33; When θ = 0 then t = 32.727

14. f ' (x) = $3x^2$-18x -12; f " (x)=6x-18; x = 3; f ' (3) = 3(9)-18(3) = -39

15.
A $\quad$ acceleration = -16; integrating twice gives s = -8t2 s(10) = – 800 feet
B. The plane's velocity is a constant; d = rt; d = (10)(50) = 500 ft



$\tan\theta = \frac{y}{500};\quad y = -800 - 16t - -8t^2;\ \sec^2\theta\frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}; \sec^2\theta\frac{d\theta}{dt} = \frac{16t - 16}{500}$

When they reach the same height a = 0 and t = 11.0499 seconds.
Da/dt = -.322 radians

- 0.322(-800 – 500)= 418.6