Calculus Team *Vero Beach Invitational* *January 24, 2004*

Solutions page 1

1. f'(x) = 3(x + 3)(x + 1)   f'(x)

+ 0 - 0 +

-3 -1

f''(x) = 6(x + 2)   f''(x)

- 0 +

f (x) is decreasing on the interval (-3, -1) and is concave up on the interval (-2, ∞) and is decreasing and concave up on the interval *(-2, -1) or [-2, -1]*

2. $V = x^2h$ $SA = x^2 + 4xh = 256$;

$$V = x^2\left(\frac{256-x^2}{4x}\right) = 27x - \frac{x^3}{4} \quad \frac{dV}{dx} = 64 - \frac{3}{4}x^2; \quad 4(64) = 3x^2; \quad x = \frac{16\sqrt{3}}{3}; \; h = \frac{256 - \frac{256}{3}}{\frac{16}{\sqrt{3}}} = \frac{8\sqrt{3}}{3}$$

$$\frac{16\sqrt{3}}{3}, \frac{16\sqrt{3}}{3}, \frac{8\sqrt{3}}{3}$$

3. A. $2x - 8 = 0$; $x = 4$ B. $\frac{f(2)-f(0)}{2-0} = -6 \Rightarrow 2x - 8 = -6;\; 2x = 2;\; x = 1$ $\frac{4}{1} + \left(-\frac{32}{3}\right)(-16) =$

C. $\frac{1}{8-0}\int_0^8 (x^2 - 8x)dx = -\frac{32}{3}$ D. f(4) = 16-32= -16 $\cancel{\frac{-16}{4} + (1)\left(\frac{-32}{3}\right) = \frac{-44}{3}}$ $\frac{524}{3}$

4. A Integral evaluated from 2 to 2 = 0; Apply L'Hopital's Rule and Fundamental Theorem of Calculus

$$\lim_{x \to 0} \frac{\int_2^{2+x} \frac{\cos t}{t}\,dt}{x} = \lim_{x \to 0} \frac{\frac{\cos(2+x)}{(2+x)}}{1} = \frac{1}{2}\cos 2$$

B definition of a derivative -sin(π/3) = - √3/2   C = -9/3 = -3 D = 3/(4√3)

$$\frac{\left(\frac{1}{2}\cos 2\right)(-3)}{\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{3}{4\sqrt{3}}\right)} = 4\cos 2$$

5.

$$A \quad SA = 4\pi r^2 \Rightarrow \frac{dSA}{dt} = 8\pi r\frac{dr}{dt} = 8\pi(2)\left(\frac{1}{4\pi}\right) = 4$$

$$B \quad V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow 4 = 4\pi(4)\frac{dr}{dt}; \frac{dr}{dt} = \frac{1}{4\pi}$$

$$C \quad V = \frac{4}{3}\pi(8) = \frac{32\pi}{3} \qquad D \quad SA = 4\pi(4) = 16\pi$$

$$\frac{ABC}{D} = \frac{(4)\left(\frac{1}{4\pi}\right)\left(\frac{32\pi}{3}\right)}{(16\pi)} = \frac{2}{3\pi}$$

6. Using disc method $\pi\int_{-4}^{4}\left(\frac{\sqrt{144-16x^2}}{3}\right)^2 dx-\pi\int_{-3}^{3}\left(\frac{\sqrt{144-9y^2}}{3}\right)^2 dy=64\pi-48\pi=16\pi$

$$f(x)=\frac{(3x^3-2x^2)+(-12x+8)}{(2x^3+5x^2)+(-8x-20)}=\frac{(3x-2)(x^2-4)}{(2x+5)(x^2-4)}$$

7. To find asymptotes look at $\frac{3x-2}{2x+5}$; Vertical Asymptotes $x = -5/2$

Horizontal asymptotes $y = 3/2$

8. A = Total area = $\int_0^1 (3t^2-6t+9)dt-\int_1^4 (3t^2-6t+9)dt+\int_4^5 (3t^2-6t+9)dt=28$

B = 24; Max velocity occurs at endpoint (5, 24) A - B = 4

V = sum of areas of squares from points -3 to 3 times dx

9. $4\int_{-3}^{3}[\sqrt{9-x^2}]^2 dx=36\cdot 4=144$

10. $3x^2+3y^2\frac{dy}{dx}=2y+2x\frac{dy}{dx}-3$; plug point for x and y and solve for $\frac{dy}{dx}$;

$A=-1$; B = slope of normal, perpendicular to tangent $=+1$ A - 3B $= -1 + -3 = -4$

11. A. Midpoint 6[150 + 225 + 240 + 176] = 4746
B. Trapezoid 24/(2(3)) [134 + 2(150 +180 + 225 +360 +240 =190 +176) + 134] = 4965 A - B = 4965-4746=219

12. g'(3) = f (3) = 2 point on tangent line is (3, g(3)) g(3) = amount of accumulated area from x = 2 to x = 3

$8x - 4y - 20 + \pi = 0$

13. $k(\sin^{-1}x-\sin^{-1}x)|_{-0.5}^{0.5}=\pi$ $k\left(\frac{\pi}{6}-\frac{\pi}{6}\right)=\pi$ $k=3$

14. f ' (x) = $3x^2 - 18x - 8$ f '' (x) = 6x – 18   0 = 6x – 18   x = 3   f '(3) = - 35

15. $SA=6e^2\Rightarrow\frac{d(SA)}{dt}12e\frac{de}{dt}\Rightarrow\frac{d(SA)}{dt}=\pm12(144)(.004)=6.912$