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Calculus Individual
Vero Beach Invitational
January 24, 2004

1. For what values of x is the curve, defined by the function $f(x)=4x^2+\frac{1}{2}x+5+\frac{4}{x}$, concave down?

A. [-1,0] B. $(-\infty,-1)\cup(0,\infty)$ C. $(-\infty,-1]$ D. (-1, 0) E. NOTA

2. Let f(x) and g(x) be differentiable functions with the following properties:

x	f(x)	f '(x)	g(x)	g '(x)
4	-2	-3	2	5
-1	4	-0.4	4	7

If h(x) = g(f(x)), find h ' (- 1).

A. 12 B. -12 C. – 2.8 D. – 2 E. NOTA

3. $\lim_{x\to 0} \frac{\sin 4x}{3x} =$

A. 3/4 B. 4/3 C. 1/4 D. 3 E. NOTA

4. If $\int_1^{12} |cx-6|dx = \frac{73}{3}$ where $1 < \frac{6}{c} < 12$, then c =

A. $\frac{10}{3}$ B. $\frac{4}{3}$ C. $\frac{3}{2}$ D. $\frac{2}{3}$ E. NOTA

5. If $F(x)=\int_0^{x^2}\frac{\tan(\pi t)}{(1+t)}dt$, then $F'(\frac{3}{2})=$

A. $\frac{12}{13}$ B. $\frac{8}{13}$ C. $\frac{4}{13}$ D. $\frac{3}{13}$ E. NOTA

6. A metallic right circular cylinder is continuously heated is expanding. If the height is 4 times the radius, what is the rate of change of the radius, in inches/sec, when the volume is increasing at the rate of 24π inches^3/sec and the radius is 1.5 inches?

A. $\frac{4}{9}$ B. $\frac{2}{9}$ C. $\frac{8}{9}$ D. $\frac{11}{9}$ E. NOTA

7. Find the average value of $f(x) = x^{-2}(1-x^{-1})$ on the interval [2,4].

A. $\frac{1}{64}$ B. $\frac{1}{32}$ C. $\frac{5}{64}$ D. $\frac{5}{32}$ E. NOTA

8. $\lim_{x\to\frac{\pi}{3}}\frac{\tan x-\sqrt{3}}{x-\frac{\pi}{3}}=$

A. 4 B. $\frac{4}{3}$ C. $\frac{3}{4}$ D. $\sqrt{3}$ E. NOTA

9. If $f(x)$ is a continuous function such that $\int_9^1 f(x)dx=-4$ and $\int_1^5 f(x)dx=12$ then evaluate $\int_5^9(3f(x)+6)dx$.

A. - 12 B. 0 C. 48 D. 72 E. NOTA

10. Find the maximum value of the rate of change of $f(x) = -3x^3 + 9x^2 + 52$.

A. 2 B. 9 C. 58 D. 109 E. NOTA

11. If $f(x) = x^4 - 2x^2 + 120$, find the product of x-coordinates at which the tangent line to the curve passes through the origin.

A. - 40 B. –20/3 C. - 4 D. 0 E. NOTA

12. Find the volume of the largest cylinder that can be inscribed in a sphere when the radius of the sphere is 16.

A. $\frac{16384\pi\sqrt{3}}{9}$ B. $\frac{8192\pi\sqrt{3}}{9}$ C. $\frac{256\pi\sqrt{3}}{9}$ D. 4952.881 E. NOTA

13. If $f(x) = 3^{\sin x}$, then $f'\left(\frac{\pi}{6}\right) =$

A. $(3)^{\frac{\sqrt{3}}{2}}$ B. $1.5\ln\sqrt{3}$ C. 3 D. $3\ln\sqrt{3}$ E. NOTA

14. While delivering his presents on Christmas Eve, Santa who is 6 feet tall, not the 20 feet some kids think, was walking toward a house that had a light 24 feet above the ground. If he was magically walking toward the house at the rate of 36 ft/sec (He did have a lot of distance to cover in a limited amount of time.), what was the rate of change of the length of his shadow in ft/sec, when he was 21 feet from the house?

A. –12 B. – 4 C. 4 D. 12 E. NOTA

15. The velocity of a particle moving on a line at time t is $v(t) = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ feet per second. How many feet did the particle travel from t = 1 to t = 9 seconds?

A. 536 B. 496 C. 492 D. 248 E. NOTA

16. If $\int_{-7}^{7} (\sin(3x) - 4x^5 - k)dx = 28,$ then k =

A. 28/5 B. 2 C. 0 D. - 4 E. NOTA

17. The equation of the normal line to the curve $\cos(x + y) = 3y$ at the point where $x = \frac{\pi}{2}$ is

A. $2x - y = \pi$ B. $4x - y = 2\pi$ C. $6x - 2y = 3\pi$
D. $2x + y = \pi$ E. NOTA

18. $\lim_{x \to 0} (1+x)^{\frac{5}{x}} =$

A. e B. 5 C. 5e D. e^5 E. NOTA

19. $\lim_{n \to \infty} \left[\frac{3}{n} \sum_{i=1}^{n} \left(\left(2 + \frac{3i}{n} \right)^2 - 3 \right) \right] =$

A. $\frac{88}{3}$ B. 30 C. $\frac{95}{3}$ D. $\frac{97}{3}$ E. NOTA

20 A particle moves along the x-axis. Find the average value of the velocity on the closed interval [1,4], when $v(t) = 2t^3 - 4t^2 + 3t + 2$.

A. 22 B. 24 C. 33 D. 67 E. NOTA

21. For a certain curve, $\frac{dy}{dx} = \sqrt{3 + xy + 3x + y}$. The curve passes through the points (-1,1) and (8, b). Find the value of b.

A. 8 B. 22 C. 118 D. 121 E. NOTA

22. If $f'(x) = \frac{f(x)}{x+3}$ and f(0) = 2, find f ' (8).

A. 2/3 B. 4/3 C. 21/3 D. 25/3 E. NOTA

23. Find the domain of the graph of $y = \ln\left(\sqrt{5 + 4x - x^2}\right)$.

A. $(-\infty, -1) \cup (5, \infty)$ B. [-1, 5] C. [0, 5] D. $(5, \infty)$ E. NOTA

24. $\frac{d}{dx}(\cos(2x)) =$

A. $-2\sin(2x)$ B. $-\cos(2x)$ C. $2\sin(2x)$ D. $-\sin(2x)$ E. NOTA

25. If $f(x) = f(x) = 3^{\cos(2x)}$, then f ' (x) =

A. $3^{\cos(2x)}\sin(2x)(\ln\frac{1}{9})$ B. $3^{\cos(2x)}2(\ln 3)$ C. $(2)3^{\cos(2x)}\sin(2x)(\ln 2)$

D. $3^{\cos(2x)}\sin(2x)(\ln 9)$ E. NOTA

26. The half-life of carbon-14 is about 5730 years. If 2 grams of carbon-14 are present in a sample, how much will be present in 10000 years?

A. 0.667 B. 0.653 C. 0.602 D.0.597 E. NOTA

27. Given that f(2) = 5, f ′ (2) = 0 and f ″(2) = - 4, then the point (2,5) could be a

A. relative minimum B. point of inflection C. terrace point
D. relative maximum E. NOTA

28. An Italian restaurant has determined that the monthly demand for its pizza is $\frac{50000-x}{18000}$, where x is the price per pizza. Find the increase in revenue per pizza, to the nearest penny, for the monthly sales of 15000 pizza.

A. 1.05 B. 1.09 C. 1.11 D. 1.13 E. NOTA

29. $\int 3x^2 dx =$

A. 3x + C B. x^3 + C C. $1.5x^2$ + C D. 6x + C E. NOTA

30. $\lim_{x \to 2} \frac{x^3-8}{x-2} =$

A. 2 B. 4 C. 8 D. 12 E. NOTA