8

Florida Invitational--Vero Beach
Answers

Algebra 2 Team

1. $\frac{8}{5}$ hour or 96 minutes.

2. $\frac{243\pi}{4}$ yds^2

3. 1

4. {k∈Reals, except k≠ $-\frac{17}{28}$}

5. -42

6. 36

7. 13

8. 205

9. 160

10. 60

11. $-(1 + \sqrt[4]{2} + \sqrt{2} + \sqrt[4]{8})$ or $-(1 + 2^{\frac{1}{4}} + 2^{\frac{1}{2}} + 2^{\frac{3}{4}})$

12. x = - 2 or x = 1

13. $\frac{9!}{2}$ or 181440

14. 20

15. 3879

Solutions: Alg 2 Team

Florida Invitational
February 7, 1998

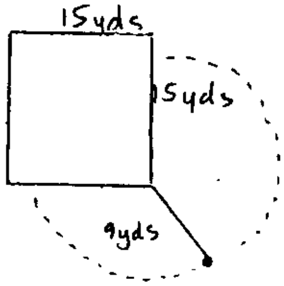
1. Mary's rate: $1\cdot(3+M)=6$
$3+M=6$
$M = 3$ mph.

Ryan + Joe's rate: $(R+J-3)\,8=6$
$8R+8J-24=6$
$8R+8J=30$
$4R+4J=15$
$R+J=\frac{15}{4}$

$R+J+M$'s rate $=\frac{15}{4}+3$

R, M + J traveling upstream $\left(\frac{15}{4}+3-3\right)x=6$
$\frac{15}{4}x=6$
$x = 8/5$ hr. OR 96 min

2.



Grazing area = $\frac{3}{4}$ circle of radius 9yds
$A=\frac{3}{4}\pi\, 9^2 = \frac{3}{4}\pi\, 81 = \frac{243\pi}{4}$ yds^2

3. roots of $4x^2+7x-5=0$: $x=\frac{-7\pm\sqrt{129}}{8}$ by quadratic formula : add 1
eccentricity of a parabola $=1$: add 0
$\sum_{x=1}^{\infty}\left(\frac{3}{n}\right)^x=1$ when $n=5$ $\quad \frac{3}{5}+\left(\frac{3}{5}\right)^2+\dots+\left(\frac{3}{5}\right)^\infty$ $\quad S_\infty : \frac{a}{1-r}=\frac{\frac{3}{5}}{1-\frac{3}{5}}=\frac{\frac{3}{5}}{\frac{2}{5}}=\frac{3}{2}$: add 0
$\frac{x^2}{9}+\frac{y^2}{9}=1 \;=\; x^2+y^2=9 \quad r=3 \quad c=\pi d = 6\pi$: add -6
Volume of a sphere $=\frac{4}{3}\pi r^3$, hemisphere $=\frac{2}{3}\pi r^3$ when $r=3$ $\;\frac{2}{3}\pi\, 27 = 18\pi$: add 6
$1+0+0+-6+6=1$

4. $\left.\begin{array}{l} 2y-x=10 \\ \frac{7}{5}kx-\frac{14}{5}ky=17 \end{array}\right\}$ are inconsistent when the lines are ∥ i.e. no solutions exist

slope of $2y-x=10$ is $1/2$
slope of $\frac{7}{5}kx-\frac{14}{5}ky=17$
$-\frac{14}{5}ky=-\frac{7}{5}kx+17$
$y=\left(\frac{-7k}{5}\right)\left(\frac{-5}{14k}\right)x+17\left(\frac{-5}{14k}\right)$
$y=\frac{x}{2}-\frac{85}{14k}$
slope $=1/2$ ∴ lines are always ∥ unless $10=\frac{-85}{14k}$
in which case they are the same line
$k\neq-\frac{17}{28}$

$\left\{ k\in\mathbb{R} \text{ except } k\neq-\frac{17}{28}\right\}$

5. $A = 8 + {}^{-}8 = 0$

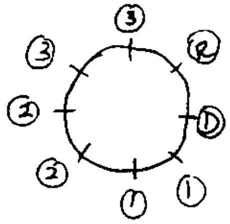
$B = 10101_2 = 16 + 4 + 1 = 21$

$C = \ln e = 1$

$D = f^{-1}g(3)$; $f^{-1}(x) = \frac{x-3}{2}$ $g(3) = 6$

$f^{-1}(6) = \frac{6-3}{2} = \frac{3}{2}$

$$\frac{A+B}{C-D} = \frac{21}{1-\frac{3}{2}} = \frac{21}{-\frac{1}{2}} = -42$$

6. 

David sits somewhere first, Rebecca sits to his right. There are 3 choices for the next position which must be a boy, 3 for the next which must be a girl, 2 for the next, 2 for the next, then 1 for the last 2 positions.

$\therefore$ $3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 = 36$ possible seating arrangements

7. $w = \frac{1}{2}(x+y+z)$ or $w = \frac{1}{2}(60-w)$ $w + x + y + z = 60$

$x = \frac{1}{3}(w+y+z)$ or $x = \frac{1}{3}(60-x)$

$y = \frac{1}{4}(w+x+z)$ or $y = \frac{1}{4}(60-y)$

$z = ?$

$w = 30 - \frac{w}{2}$ $x = 20 - \frac{x}{3}$ $y = 15 - \frac{y}{4}$

$\frac{3}{2}w = 30$ $\frac{4}{3}x = 20$ $\frac{5}{4}y = 15$

$w = 20$ $x = 15$ $y = 12$

$20 + 15 + 12 = 47$ $\therefore$ $47 + z = 60$ $\therefore$ $z = 13$

8. $\frac{771 + x}{16} = 61$

$771 + x = 976$

$x = 205$ yds.

9. 1 2 ~~3~~ 4 5 6 7 8 9 10 11 12

1: 2nd, 3: ~~2nd~~, 5: 2nd, 7: 2nd, 9: 2nd, 10: 1st, 11: 2nd 1st, 12: 1st

They must've missed question 11 since they answered in the first minute – and the 1st correct answer was in the 2nd min. The max. would be $10 \times 16 = 160$

10. For the maximum seating – start seating in first seat, each row has 8 people

11th row – 7
10th row – 7
9 – 6
8 – 6
7 – 5
6 – 5
5 – 4
4 – 4
3 – 3
2 – 3
1 – 2

60 people

11. $t = \dfrac{1}{1-\sqrt[4]{2}}$ $\quad \dfrac{1}{1-\sqrt[4]{2}} \cdot \dfrac{1+\sqrt[4]{2}}{1+\sqrt[4]{2}} = \dfrac{1+\sqrt[4]{2}}{1-\sqrt{2}} \cdot \dfrac{1+\sqrt{2}}{1+\sqrt{2}} = \dfrac{1+\sqrt[4]{2}+\sqrt{2}+\sqrt[4]{8}}{1-2} =$

$-\left(1+\sqrt[4]{2}+\sqrt{2}+\sqrt[4]{8}\right)$ or $-\left(1+2^{1/4}+2^{1/2}+2^{3/4}\right)$

12. $3^{2x+2} - 3^{x+3} - 3^{x} + 3 = 0$

$3^{2x} \cdot 3^{2} - 3^{x} \cdot 3^{3} - 3^{x} + 3 = 0$

$9 \cdot 3^{2x} - 3^{x}(27+1) + 3 = 0 \quad 9 \cdot 3^{2x} - 28 \cdot 3^{x} + 3 = 0$

let $y = 3^{x}$ then $9y^{2} - 28y + 3 = 0$

$(9y-1)(y-3) = 0$

$y = \frac{1}{9}$ or $y = 3$

$3^{x} = \frac{1}{9} \qquad 3^{x} = 3$

$3^{x} = 3^{-2} \qquad 3^{x} = 3^{1}$

$x = -2, \qquad x = 1$

Both solutions check:

$3^{-2} - 3^{1} - 3^{-2} + 3 = 0$ ✓

$3^{4} - 3^{4} - 3 + 3 = 0$ ✓

13. $\dfrac{(n-1)!}{2} = \dfrac{9!}{2}$ or 181,440

14. length $= l$ width $= w$

$lw = (l+\frac{5}{2})(w-\frac{2}{3}) \quad lw = (l-\frac{5}{2})(w+\frac{4}{3})$

$\therefore -\frac{2}{3}l + \frac{5}{2}w = \frac{10}{6}$ and $\frac{4}{3}l - \frac{5}{2}w = \frac{20}{6}$

$l = \frac{15}{2}, \; w = \frac{8}{3} \quad \therefore \quad lw = 20$

15. $y = 10x + 10$ $\quad x =$ time ; $y =$ population

$5x - 3y = -1000$ $\quad x =$ population $y =$ food.

to find pop. That consumes 100,000 units of food

$5x - 300{,}000 = -1000$

$5x = 299{,}000$

$x = 59{,}800$

to find what year that pop is reached: pop is explained by $y = 10x + 10$ after 1900 when pop is 40,000 so $59{,}800 - 40{,}000 = 19{,}800$

$19{,}800 = 10x + 10$

$19790 = 10x$

$1979 = x$ years

$1979 + 1900 = 3879.$