Algebra I Team Solutions
Vero Beach Invitational March 19, 2005

1. 10 seconds The arrow is traveling at 40 feet per second, so it will travel a total distance of 40X, where X is the number of seconds the flight takes. In that time, the tortoise will be 200 + 20X feet away from Xeno. $40X = 20X + 200$, so $20X = 200$. Thus, $X = 10$ seconds.

2. 238 The largest prime number less than 100 is 97. Also, $3.14 < \pi < 22/7$ and 0! times 1! = 1, which is a natural number. Therefore the instructions read (97 + 5)/3, which equals 34. The least common multiple of 34 and 119 is 238.

3. Mrs. Hiller; $11689 Thomas celebrates his birthday once a year, for eight years. Each time, he gets $1460, for a total of $11680. Mrs. Hiller has her birthday twice every eight years, as there are two leap years in every eight-year-period. As she receives $5844.50 every time she has a birthday, after two birthdays she's amassed $11689. Eight years = 6(365) + 2(366), so in eight years Alton has 2190 + 732 = 2922 birthdays. He gets $4 for each one, so he gets $11688 in total. Mrs. Hiller gets the most money in an eight-year period.

4. 1 Simplifying, $\dfrac{\dfrac{a(a+3)-3(a-3)}{(a-3)(a+3)}}{\dfrac{3(a+3)+a(a-3)}{(a-3)(a+3)}} = \dfrac{\dfrac{a^2+9}{a^2-9}}{\dfrac{a^2+9}{a^2-9}} = 1$

5. 110 $W = (11)^2$, $X = (11)^5$, $Y = (11)^4$, and $Z = (11)^{-1}$. So, $WY/XZ = (11)^6/(11)^4 = 121$. 121 – 11 = 110.

6. 32 (5! - 4! - 3! - 2! - 1!) = 120 – 24 – 6 – 2 – 1 = 87. 87's prime factorization is (3)(29), and 3 + 29 = 32.

7. I and III $4.75 = 3h + x$, and $4.25 = h + 3x$. Solving the system of equations gives h, the price of a hotdog, a value of $1.25, and x, the price of a soda, a value of $1. Another system is set up, using the equations $1 = c + y$, and $2.25 = 3c + 2y$. This system gives c, the price of a candy bar, as $0.25, and y, the price of a box of popcorn, as $0.75. Systems I and III yield the correct prices of soda and popcorn.

8. 25 At the start of the day, the bowl has X dates in it. After breakfast, it has (2/3)(X – 1). After lunch, it has (2/3)[(2/3)(X – 1) – 1]. After dinner, it has (2/3){(2/3)[(2/3)(X – 1) – 1] -1}, which can be simplified to (8X – 38)/27. Since there are 6 dates left after dinner, (8X – 38)/27 = 6. Therefore, X = 25.

9. .999316 In a twelve year period, there are 3 leap years- thus, there are 3 days in which it does not rain in Brithosomania. There are 9(365) + 3(366) = 4383 days in twelve years. (4383 – 3)/4383 = 0.999316.

10. 99x $x = (2)(25) = 50$. $1 + 2 + 3 + 4 + \ldots 96 + 97 + 98 + 99 = 49(100) + 50 = 98(50) + 50$. Therefore, $1 + 2 + 3 + 4 + \ldots 96 + 97 + 98 + 99 = 99x$.

11. $9x^2 – 4x – 40$ Using OFF, $A = 3x + x^2 + x^2 = 2x^2 + 3x$. Using FILL, $B = x^2 – 5x – 5 – 5 = x^2 – 5x – 10$. Using LOL, $C = -6 – x – 6 = -x – 12$. Using FOOL, $D = x^2 + 2x + 2x – 4 = x^2 + 4x – 4$. Using OIL, $E = -2x – 6x – 12 = -4x – 12$. Using FOOF, $F = x^2 + x + x + x^2 = 2x^2 + 2x$. Using OF, $G = -3x + x^2 = x^2$

– 3x. Using OFFILL, $H = -x + x^2 + x^2 + x - 1 - 1 = 2x^2 - 2$. $A + B + C + D + E + F + G + H = 2x^2 + 3x + x^2 - 5x - 10 + -x - 12 + x^2 + 4x - 4 + -4x - 12 + 2x^2 + 2x + x^2 - 3x + 2x^2 - 2 = 9x^2 - 4x - 40$.

12. 1337 $W = 345_{(wt-f)} = 345_7 = 180_{10}$. E = (asl)/13 ÷ 15 = (26)(8)(15) / [(13)(15)] = 16. N = -91o!g/m! = (-91)(12!)(20)/(14!) = -1820 / [(14)(13)] = -10. D = 17/16. (p + W)(N + ED) = (11 + 180)(-10 + (16)(17/16)) = (191)(7) = 1337.

13. 3840 $10! \div 9! \bullet 8! \div 7! \bullet 6! \div 5! \bullet 4! \div 3! \bullet 2! \div 1! = 10 \bullet 8 \bullet 6 \bullet 4 \bullet 2 = 3840$.

14. 2 hours and 16 minutes. $18 \bullet 4 > 56 > 18 \bullet 3$, so Moqueef will have to earn 4 eighteen minute tv sessions. $4 \bullet 34 = 136$, so Moqueef has to spend 136 minutes doing his homework. 136 minutes is 2 hours and 16 minutes.

15. The smallest prime number is 2. Its square is 4. The smallest prime number larger than 4 is 5. Twice the square of 5 is 50. The largest prime number less than 50 is 47. The cube root of 47 is $\sqrt[3]{47}$