Algebra I Individual Solutions
Vero Beach Invitational march 19, 2005

1. B. 25 With each term in the sequence, the amount added to the previous term is increased by two. -5 + **2** = -3. -3 + **4** = 1. 1 + **6** = 7. 7 + **8** = 15. Therefore, the next term will be the previous term (15) plus two more than the amounted added the time previously (8 + 2 = 10). 15 + 10 = 25, so 25 fills in the gap in the sequence. This pattern continues to the seventh term, as 25 + **12** = 37.

2. E. All of the answers are equivalent. 1 = 1 and 1! = 1. The polynomial expression ($12x^2 + 16x - 35$) can be factored as $(6x - 7)(2x + 5)$. $(2x + 5)^{-1}$ is equal to $1/(2x + 5)$. Therefore the expression reads $[(6x - 7)(2x + 5)] / [(6x - 7)(2x + 5)]$. Assuming nonzero denominators as the instructions read, choice C is equivalent to 1. If choice D ($.\overline{9}$) is set equal to x, then $10x = 9.\overline{9}$. $10x - x = 9x$, and $9.\overline{9} - .\overline{9} = 9$. Thus $9x = 9$, so x (choice D) equals 1. All answers are equivalent, as all answers equal 1.

3. C. 12.5 A system of equations is given. Using x as the weight of a monkey and y as the weight of an October, $x + 7y = 3x + 4y + 60$. The next equation given is $22x + 6y = y - 14$. These equations can be simplified to $2x - 3y = -60$ and $22x + 5y = -14$. This system of equations gives -4.5 as the value of x, and 17 as the value of y. (Remember, on Blarg weight can be negative). The weight of one monkey and one October equals x + y, or -4.5 + 17, which equals 12.5.

4. B. 6 a☺b = (a + b), c⚡ = (c – 2)(c + 2), d 💣e = **d!/e!**, and f☯ equals the as sum of f's digits. Thus (((2☺3)⚡) 💣19) ☯ equals (((2 + 3)⚡) 💣19) ☯ equals ((5⚡) 💣19) ☯ equals (((5 – 2)(5 + 2)) 💣19) ☯ equals (((3)(7)) 💣19) ☯ equals (21💣19) ☯ equals (21!/19!) ☯ equals ((21)(20)) ☯ equals ((21)(20)) ☯ equals 420☯ equals the sum of the digits of 420 equals 4 +2 equals 6.

5. D. 9986 $f(x) = (x^2)(x - 3)$, $g(x) = (x - 3)(x + 3)$, and $h(x) = (3x + 2)$. $h(f(g(h(1)))) = h(f(g(3 + 2))) = h(f(g(5))) = h(f((5 - 3)(5 + 3))) = h(f((2)(8))) = h(f(16)) = h((16^2)(16 - 3)) = h((256)(13)) = h(3328) = 3(3328) + 2 = 9984 + 2 = 9986$.

6. A. 2½ In cooperative working, total time/first person alone + total time/second person alone = 1.

T/8 + T/3.5 = 1
7T/56 + 16T/56 = 1
23T/56 = 1
23T = 56
T = 56/23 ≈ 2.4347
Rounded to the nearest half year, T = 2½ years

7.D. 673/90 If $7.4\overline{7}$ is set equal to x, $10x = 74.\overline{7}$ and $100x = 747.\overline{7}$. $100x - 10x = 90x$, and $747.\overline{7} - 74.\overline{7} = 673$. $90x = 673$, therefore $x = 673/90$. The ratio of the denominator to the numerator is 90/673, and its reciprocal is 673/90.

8. A. None Distributing shows that $5n - 15 + 2n^2 - 39 - 3n = 144 + 12n + 2n + 2n^2 - 12n$. This simplifies to $2n^2 + 2n - 54 = 2n^2 + 2n + 144$. Any real value of n placed into the equation yields the result -39 = 144, which is not true. Thus, there are no distinct real values of n.

9. C. 7/400 The race is 26.2 miles long, which is 65.5 0.4 mile sections. For each 0.4 mile section Thomas runs, he loses 0.015 gallons of milk, so he loses (65.5)(0.015) = 0.9825 gallons of milk in total. 1 gallon – 0.9825 gallons = 0.0175 gallons. 0.0175 expressed as a simplified fraction = 7/400.

10. B. $O(5T + \frac{N}{U})$

$$\frac{\frac{D}{O} - \frac{N}{U}}{T} = 5$$

$$\frac{D}{O} - \frac{N}{U} = 5T$$

$$\frac{D}{O} = 5T + \frac{N}{U}$$

$$D = O(5T + \frac{N}{U})$$

11. C A diagram such as the following is used to solve the problem.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | | | | | | |
| Sengupta | | | | | | |
| Polk | | | | | | |
| Amy | | | | | | |
| Weir | | | | | | |
| Kessler | | | | | | |

S. does not catch 2 frogs:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | | | | | | |
| Sengupta | | X | | | | |
| Polk | | | | | | |
| Amy | | | | | | |
| Weir | | | | | | |
| Kessler | | | | | | |

W. catches more frogs than P. but less than J. This means W. cannot catch 1 frog, P. cannot catch 6, W. cannot catch 6, and J. cannot catch 1:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | | | | | |
| Sengupta | | X | | | | |
| Polk | | | | | | X |
| Amy | | | | | | |
| Weir | X | | | | | X |
| Kessler | | | | | | |

A. catches 4 more than P. Thus, A. cannot catch 1, 2, 3, or 4, and P. cannot catch 3, 4, 5, or 6:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | | | | | |
| Sengupta | | X | | | | |
| Polk | | | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | | | | X |
| Kessler | | | | | | |

S. catches more than K., but less than J. Thus, S. cannot catch 1, K. cannot catch 6, J. cannot catch 1, and S. cannot catch 6:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| Jerkins | X | | | | | |
| Sengupta | X | X | | | | X |
| Polk | | | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | | | | X |
| Kessler | | | | | | X |

Also, as S < J, and S. catches either 3, 4, or 5, J. catches either 4, 5, or 6:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | | |
| Sengupta | X | X | | | | X |
| Polk | | | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | | | | X |
| Kessler | | | | | | X |

The person who catches two frogs is later alphabetically than the person who catches one frog. Either P., W., or K. caught two frogs, and either P. or K. caught one frog. K. is not alphabetically later than P. or W., so K. cannot catch 2 frogs:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | | |
| Sengupta | X | X | | | | X |
| Polk | | | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | | | | X |
| Kessler | | X | | | | X |

K. catches three frogs, therefore K. cannot catch 1, 2, 4, 5, or 6 frogs, and no one else can catch 3:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | | |
| Sengupta | X | X | X | | | X |
| Polk | | | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | X | | | X |
| Kessler | X | X | • | X | X | X |

At this point, it can be seen that P. is the only person who could have caught 1 frog. Therefore P. cannot catch 2, 3, 4, 5, or 6 frogs:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | | |
| Sengupta | X | X | X | | | X |
| Polk | • | X | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | | X | | | X |
| Kessler | X | X | • | X | X | X |

This means W. had to catch 2 frogs (and therefore not 1, 3, 4, 5, or 6):

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | | |
| Sengupta | X | X | X | | | X |
| Polk | • | X | X | X | X | X |
| Amy | X | X | X | X | | |
| Weir | X | • | X | X | X | X |
| Kessler | X | X | • | X | X | X |

Reading back through the clues, A. = P. + 4. As P. = 1, A. = 5, not 6, and no one else catches 5 frogs:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | | X | |
| Sengupta | X | X | X | | X | X |
| Polk | • | X | X | X | X | X |
| Amy | X | X | X | X | • | X |
| Weir | X | • | X | X | X | X |
| Kessler | X | X | • | X | X | X |

This means J. catches 6 frogs, not 4:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | X | X | • |
| Sengupta | X | X | X | | X | X |
| Polk | • | X | X | X | X | X |
| Amy | X | X | X | X | • | X |
| Weir | X | • | X | X | X | X |
| Kessler | X | X | • | X | X | X |

Which means that S. catches 4 frogs:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Jerkins | X | X | X | X | X | • |
| Sengupta | X | X | X | • | X | X |
| Polk | • | X | X | X | X | X |
| Amy | X | X | X | X | • | X |
| Weir | X | • | X | X | X | X |
| Kessler | X | X | • | X | X | X |

12. B. II only I is not true, as $|n| = 0$ when $n = 0$, and 0 is not positive. II is always true- if x is the length of a triangle's side, x must be a positive number, as a triangle cannot have a side measuring a negative length or measuring zero. III is not true, as $(-\sqrt{17})^2$ is 17, and $-\sqrt{17}$ is negative.

13. B. A difference of squares;Hans lines up his soldiers in seven rows by seven columns. $(7)(7) = 49$. He then adds a row and takes away a column, with room for forty-eight soldiers. $(8)(6) = 48$. This can be written as $(7 + 1)(7 - 1) = 48$, or $(7 + 1)(7 - 1) = (7^2 - 1)$, which is a difference of squares.

14. D. 6 Substituting 3 in for z, the system of equations $3x + y + 3 = 22$ and $2x - y - 12 = -1$ is given. This is equivalent to the equations $3x + y = 19$ and $2x - y = 11$. When solved, 1 is given as the value of y, and 6 is given as the value of x.

15. D. $x^4 + x^2 + 4x$ $9 = (\sqrt{3})^4$, $\sqrt{(75)} = \sqrt{(25)}\sqrt{3} = 5\sqrt{3}$, and $3 = (\sqrt{3})^2$. Plugging $\sqrt{3}$ in for x, the expression reads $x^4 + 5x - x + x^2$, which, when rearranged and simplified, reads $x^4 + x^2 + 4x$.

16. C. 3/2 The probability of drawing a blue marble is 2/5, and the probability of drawing a red marble is 3/5. Thus, for every two blue marbles, there are three red marbles. So for every blue marble there are 3/2 of a red marble.

17. A. $y = (2!)^3$ $y = 8$. $(2!)^3 = 2^3 = 8$. The others do not equal 8. $2^{(3!)} = 2^6 = 64$. $(2!)^{(3!)} = 2^6 = 64$. $8! - 7! = 40320 - 5040 = 35280$. Only $(2!)^3 = 8 = y$.

18. B. 4 $(x - 2)^2 - 2(x - 2) + 1 = 0$. Thus, $y^2 - 2y + 1 = 0$, so $y = 1$. $1 = x - 2$, so $x = 3$. $x + y = 1 + 3 = 4$.

19. A 0 Solving for $y$: $z = \frac{1+y}{1-y}, z - yz = 1 + y, z - 1 = y + yz,$ $z - 1 = y(1+z), y = \frac{z-1}{1+z}$ . $x + y = \frac{1-z}{1+z} + \frac{z-1}{1+z} = \frac{0}{1+z} = 0.$

20. D. 496 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30 + 31 = 496.

21. B. 15.5 In the month of January, Sam eats 31(5/12 of an ice cream sandwich), or 155/12 ice cream sandwiches. Also, he eats 31(11/12 of a sock), or 341/12 socks. 341/12 – 155/12 = 186/12 = 15.5

22. E. $1.95 2.50 = 7H + 2S + .40, and 2.50 = H + 6S + .20. These equations can be simplified to 7H + 2S = 2.1 and H + 6S = 2.3. Solving the system of equation gives a value of .2 for H and .35 for S. Thus a hotdog costs 20 cents, and a soda costs 35 cents. This means one hotdog and one soda cost 55 cents total. Paying $2.50 would give $1.95 change.

23. B. 24 2Q = C/2, so C = 4Q. Q = 2J, so C = 8J. 3J = 9H, so J = 3H. C = 8(3H) = 24H. 24H in 1C.

24. C. x, where $x^2 = 4$. √2 has only one value, as √2 signifies only a positive value. $2^2$ has only one value, that of 4. If $x^2 = 4$, x can be one of two possible values, 2 or -2. x where $x^3 = 4$ has only a single, positive value.

25. D. 25 milesThe bikes will collide exactly one hour after they begin to move. The fly is flying at a constant rate of 25 miles per hour, so in the one hour the bikes are traveling, the fly will travel 25 miles.

26. A. -8/99 8/11 + 8/99 - 8/9 = 72/99 + 8/99 - 88/99 = -8/99.

27. E. 118.5 5 and 12 are relatively prime- their greatest (and only) common factor is 1. So P = 1. Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84... Multiples of 13 are 13, 26, 39, 52, 65, 78, 91...The least common multiple of 6 and 13 is 78, so Q = 78. R = (P + Q)/2 = (1 + 78)/2 = 79/2 = 39.5. 3R = 3(39.5) = 118.5.

28. D. 39 W = 13/X, and Z = 12/Y. WZ = (13 • 12)/(XY). XY = 4, so WZ = 156/4 = 39.

29. A. 36 (79 + 100 + 64 + 52 + 86 + 94 + X)/7 ≥ 73, so 79 + 100 + 64 + 52 + 86 + 94 + X ≥ 511. This means that 475 + X ≥ 511, so X ≥ 36.

30. C. 4 years; There were 30 fish originally in the school. Half the fish have been eaten by the time Drew realizes he must leave. Thus, 15 fish have been eaten. The piranhas take 3 months to eat a single fish, therefore it took them 45 months to eat 15 fish. 45 months= 3.75 years, which rounded to the nearest year is 4 years.