Answers to Pre-Calculus Individual and Team--February 24, 1996

Individual

1. D.
2. D.
3. C.
4. B.
5. A.
6. A.
7. D.
8. A.
9. A.
10. A.
11. B.
12. E. 208
13. C.
14. B.
15. D.
16. B.
17. A.
18. B.
19. E. (0,10)
20. C.
21. B.
22. D.
23. A.
24. B.
25. D.
26. C.
27. C.
28. A.
29. A.
30. C.

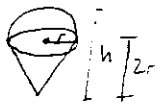
Team

1. 0
2. $\frac{4}{3}\pi r^3$
3. $\frac{\sqrt{13}}{3}$
4. $a(cg-df)$
5. (5,0) and (-4,3)
6. 2
7. 32
8. 5522
9. 0.215
10. 1,1,2,3,5,8,13,21,34,55
11. 32
12. 26.8°
13. $34,207.47
14. 120
15. 144

TEAM ROUND SOLUTIONS - PRE CALCULUS PCHS

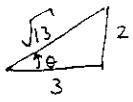
1.) $x^3 - 8 = 0$

Product of the roots taken two at a time is the coefficient of the x term divided by the coefficient of the x^3 term. In this case $\boxed{C.}$

2.) 

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \frac{1}{3}(\pi r^2)(2r)$

$V = \boxed{\frac{4}{3}\pi r^3}$

3.) 

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{3}{\sqrt{13}}} = \boxed{\frac{\sqrt{13}}{3}}$

4.) $\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b & c & d \\ e & f & g \end{vmatrix} = (cg - df)\hat{i} + (de - bg)\hat{j} + (bf - ce)\hat{k}$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = a\hat{i}\cdot[(cg-df)\hat{i} + (de-bg)\hat{j} + (bf-ce)\hat{k}]$

$= \boxed{a(cg - df)}$

5.) $x = 5 - 3y \Rightarrow (5-3y)^2 + y^2 = 25$

$25 - 30y + 10y^2 = 25$

$10y(y-3) = 0$

$y = 0 \Rightarrow x = 5$

$y = 3 \Rightarrow x = -4$

$\boxed{(5,0) \text{ and } (-4,3)}$

6.) $\frac{x^2+10x-36}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$= \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$ ← I'm leaving the denominator out of the rest.

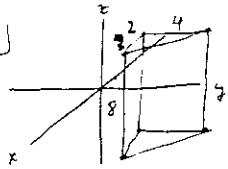
$= (A+B)x^2 + (-6A - 3B + C)x + 9A$

So $A + B = 1$

$-6A - 3B + C = 10 \Rightarrow A = -4;\ B = 5;\ C = 1$

$9A = -36$

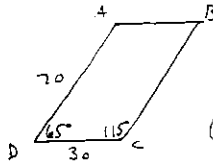
$A + B + C = \boxed{2}$

7.) 

Triangular Prism

$V = Bh = \frac{1}{2}(2\cdot 4)\cdot 8$

$= \boxed{32}$

8.) 

Cosine Law

$(AC)^2 = 70^2 + 30^2 - 2\cdot 70\cdot 30\cos 65°$

$AC \approx 63.4429$

$(BD)^2 = 70^2 + 30^2 - 2\cdot 70\cdot 30\cos 115°$

$BD \approx 87.0345$

$AC \cdot BD \approx \boxed{5522}$

9.) Area Ellipse $= \pi ab = \pi\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) = \frac{15\pi}{4}$

$P(\text{outside target}) = \frac{15 - \frac{15\pi}{4}}{15} = 1 - \frac{\pi}{4} = \boxed{0.215}$

10.) $a_1 = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right) - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right) = 1$

$a_2 = \frac{1}{\sqrt{5}}\left(\frac{6+2\sqrt{5}}{4}\right) - \frac{1}{\sqrt{5}}\left(\frac{6-2\sqrt{5}}{4}\right) = 1$

$a_3 = \frac{1}{\sqrt{5}}\left(\frac{16+8\sqrt{5}}{8}\right) - \frac{1}{\sqrt{5}}\left(\frac{16-8\sqrt{5}}{8}\right) = 2$

$a_4 = \frac{1}{\sqrt{5}}\left(\frac{56+24\sqrt{5}}{16}\right) - \frac{1}{\sqrt{5}}\left(\frac{56-24\sqrt{5}}{16}\right) = 3$

$a_5 = \frac{1}{\sqrt{5}}\left(\frac{176+80\sqrt{5}}{32}\right) - \frac{1}{\sqrt{5}}\left(\frac{176-80\sqrt{5}}{32}\right) = 5$

The pattern is 1, 1, 2, 3, 5, ... which is the Fibonacci sequence. Answer: $\boxed{1, 1, 2, 3, 5, 8, 13, 21, 34, 55}$

11.) $\frac{{}_{365}P_n}{365^n}$ = Prob. all n people have a different birthday

$n = 10 \rightarrow \sim .88$

$= 20 \rightarrow \sim .59$

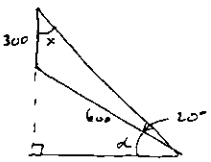
$= 30 \rightarrow \sim .29$

$= 35 \rightarrow \sim .19 \rightarrow = 31 \rightarrow \sim .27$

$= \boxed{32} \rightarrow \sim .247$

12.) Sine Law: $\frac{300}{\sin 20°} = \frac{600}{\sin x}$

$\Rightarrow x = 43.160°$



Drop a perpendicular to the horizontal.

$5\alpha + (20° + \alpha) + 90° + 43.160° = 180°$

$\boxed{\alpha = 26.8°}$

13.) $P_1 = 10000\left(1 + \frac{(.08)}{365}\right)^{365\cdot 30} = \$110202.777$

$P_2 = 10000\left(1 + \frac{(.09)}{4}\right)^{4\cdot 30} = \$144410.244$

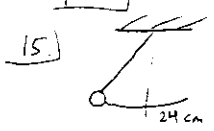
$|P_2 - P_1| = \$34,207.47$

14.) $\det A = \det\begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & +5 & -1 & 6 \\ 1 & 0 & -2 & 3 \\ -3 & 2 & 0 & -5 \end{bmatrix}$

$-(-1)\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 3 \\ -3 & 2 & -5 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ -3 & 2 & -5 \end{vmatrix}$

$= [-27 + 8 + 15 - 12 + 0] - 2[-50 - 54 + 0 - 24 + 60]$

$= \boxed{120}$

15.) 

$\frac{24}{1 - \frac{5}{6}} = \boxed{144}$