SCHS Inv. 2/24/96 P-C Individual Sol.

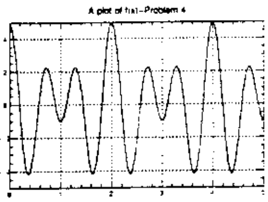
1.) $(x+1)^2 + (y+1)^2 + (z-2)^2 = 19 + 1 + 1 + 4$
$= 25 \Rightarrow r = 5$

$V = \frac{4}{3}\pi r^3 = \frac{500\pi}{3}$ $\boxed{D.}$

2.) $A = \frac{1}{2} ab \sin\theta = \frac{1}{2}(10)(10)\sin 80° \cong 49.2$ $\boxed{D.}$

3.) $\frac{|5(7) - 3(4) - 8|}{\sqrt{(5)^2 + (-3)^2}} \cong 2.57$ $\boxed{C}$

4.)



As the diagram shows, the period is 2.

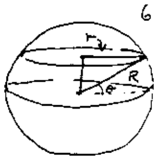
$\boxed{B.}$

5.) $5280\,ft = (60{,}470\,ft)\log\left(\frac{1015.25}{B_1}\right)$

$\Rightarrow B_1 \cong 830.34$ $\boxed{A.}$

6.) $22496 = 2^5 \cdot 19 \cdot 37$
$6 \times 2 \times 2 = 24$ $\boxed{A.}$

7.)



$R = 6370$ km $\quad r = R\cos\theta$
$\theta = 30° \Rightarrow r \cong 5516.58$ km

$\frac{20°}{360°} \cdot 2\pi r \cong 1926$ km $\boxed{D.}$

8.) The first maximum is at $x = \frac{\pi}{2}$. Then comes $x = \frac{5\pi}{2}$ and $a = \frac{9\pi}{2}$.

$f(a) = 2^{\left(-\frac{9\pi}{2}\right)} \sin\left(\frac{9\pi}{2}\right) \cong 5.55 \times 10^{-5}$ $\boxed{A.}$

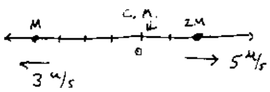
9.) $\cot 2\theta = \frac{A-C}{B} = \frac{1-(-6)}{5} = 1.4$

$\Rightarrow \theta \cong 0.31$ $\boxed{A.}$

10.) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -2 & 1 \\ 0 & -2 & 7 \end{vmatrix} = -12\hat{\imath} - 35\hat{\jmath} - 10\hat{k}$

magnitude $= \sqrt{(-12)^2 + (-35)^2 + (-10)^2}$
$\cong 38.3$ $\boxed{A.}$

11.)



The center of mass of the system is two thirds of the distance between the two particles. After 1 second, the 2M particle is at 7 while the other particle is at −7. This puts the center of mass at 7/3. So it is travelling at 7/3 units per second. $\boxed{B.}$

12.)

$n =$	0	1 ⋯ 14	15 ⋯ 79	80 ⋯ 100
$\sqrt[4]{n+1} =$	1	1 ⋯ 1	2 ⋯ 2	3 ⋯ 3

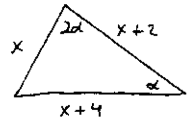
15 1's + 65 2's + 21 3's = 208 $\boxed{E.}$

13.) $\frac{e^x + e^{-x}}{2} = 2\left(\frac{e^x - e^{-x}}{2}\right) \Rightarrow \frac{3e^{-x}}{2} = \frac{e^x}{2}$

$\Rightarrow \ln 3 - x = x \Rightarrow x = \frac{\ln 3}{2}$ $\boxed{C.}$

14.) $9x^2 - 25(2)^2 - 72x - 50(2) - 106 = 0$
$9x^2 - 72x - 306 = 0$
The sum of the roots is 8 $\boxed{B.}$

15.)



Use both sine and cosine law.
Sine law:
$\frac{\sin 2\alpha}{x+4} = \frac{\sin\alpha}{x}$

$\frac{2\sin\alpha\cos\alpha}{x+4} = \frac{\sin\alpha}{x}$

$\Rightarrow \cos\alpha = \frac{x+4}{2x}$

Now use cosine law:

$x^2 = (x+2)^2 + (x+4)^2 - 2(x+2)(x+4)\left(\overset{\cos\alpha}{\frac{x+4}{2x}}\right)$

$x^2 = x^2 + 4x + 4 + x^2 + 8x + 16 - \frac{(x^2+6x+8)(x+4)}{x}$

$0 = x^2 + 12x + 20 - \frac{x^3}{x} - \frac{10x^2}{x} - \frac{32x}{x} - \frac{32}{x}$

$0 = 2x - 12 - \frac{32}{x}$

$0 = x^2 - 6x - 16$
$0 = (x-8)(x+2)$
$x = 8$ is the only "real" solution.
So the perimeter of the triangle is $8 + 10 + 12 = 30$ $\boxed{D.}$

16.) $\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta) = \frac{1-\sin^2\theta}{\cos\theta} = \cos\theta$ $\boxed{B.}$

17.)
1100101101_2
$+\,1001010101_2$
10110000010_2 $\boxed{A.}$

18.) $p \to q$
$\sim(r \wedge \sim s) \Rightarrow r \to s$
$s \to \sim q$

equivalent to $p \to q$

Given $r \to s \to \sim q \to \sim p$ $\boxed{B.}$

19.) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$

$R_1 = 10$

$0 < \frac{R_1 R_2}{R_1 + R_2} < 5 \Rightarrow 0 < \frac{10 R_2}{10 + R_2} < 5$

$\Rightarrow 10R_2 < 50 + 5R_2 \Rightarrow 0 < R_2 < 10$
$(0, 10)$ $\boxed{E.}$

20.) $\log(x^3) = (\log x)^3$
$3\log x = (\log x)^3$
$0 = \log x\,[(\log x)^2 - 3]$
$= \log x\,[(\log x) - \sqrt{3}][(\log x) + \sqrt{3}]$

$\log x = 0 \Rightarrow x = 1$
$\log x = \sqrt{3} \Rightarrow x = 10^{\sqrt{3}}$
$\log x = -\sqrt{3} \Rightarrow x = 10^{-\sqrt{3}}$ $\boxed{C.}$

21.) $(2 \times 4)\;(i \times j) = 2 \times 7$

Answer 4×7 $\boxed{B.}$

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22.) 10° in 20 seconds $\Rightarrow 30^\circ$ in 1 minute

In one minute the particle moves $1/3$ mile.

$$d = r\theta \Rightarrow \frac{1}{3} = r\frac{\pi}{6} \Rightarrow r = \frac{2}{\pi} \quad \boxed{D.}$$

23.) $\sin\theta = 4\cos\theta \Rightarrow \sin^2\theta = 16\cos^2\theta$

$\Rightarrow 1 - \cos^2\theta = 16\cos^2\theta \Rightarrow \cos\theta = \frac{\sqrt{17}}{17}$ since $\theta \in [0, \frac{\pi}{2}]$

$\Rightarrow \sin\theta = \frac{4\sqrt{17}}{17}$. So $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(\frac{4\sqrt{17}}{17}\right)\left(\frac{\sqrt{17}}{17}\right)$$

$$= \frac{8}{17} \quad \boxed{A.}$$

24.) For a conic section, eccentricity, e, shows up in the polar form in the following way...

$$r = \frac{\alpha}{1 + e\cos\theta}$$

For this case $e = \frac{3}{2}$ or a hyperbola. $\boxed{B.}$

25.) $\sum_{i=1}^{4} T_i = 4p \qquad \sum_{i=1}^{5} T_i = 5q$

Let x be the fifth test score

$$4p + x = 5q \Rightarrow x = 5q - 4p$$

$$\frac{4p + 2x}{6} = \text{new average} \Rightarrow \frac{4p + 10q - 8p}{6} = \frac{5q - 2p}{3} \quad \boxed{D.}$$

26.) $$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{2n-1} - \frac{1}{2n+1}$$

$$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \cdots\right.$$

$$= \frac{1}{2} \quad \boxed{C.}$$

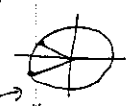
27.) $4(\sec^2 x - 1) + 2\sec^2 x + \sec x - 3 = 0$

$6\sec^2 x + \sec x - 7 = 0$

$(6\sec x + 7)(\sec x - 1) = 0$

$\sec x = 1 \Rightarrow x = 0$ or 2π

$\sec x = -\frac{7}{6} \Rightarrow \cos x = -\frac{6}{7}$



Sum of x for this solution is 2π.

So sum of all solutions is 4π. $\boxed{C.}$

28. $$\frac{2(x+h)^2 - 5(x+h) + \cancel{4} - 2x^2 + 5x \cancel{- 4}}{h}$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-5x} - 5h \cancel{-2x^2} \cancel{+5x}}{h}$$

$$= 4x - 5 + 2h \quad \boxed{A.}$$

29.) $h_1 = 4 \quad h_2 = 6 \quad h_3 = 8$

Area $= \frac{1}{2}bh \qquad \frac{1}{2}b_1h_1 = \frac{1}{2}b_2h_2 = \frac{1}{2}b_3h_3$

$$2b_1 = 3b_2 = 4b_3$$

$$r_{\text{inscribed}} = \frac{\text{Area}}{\text{semiperimeter}} = \frac{2b_1}{b_1 + \frac{2}{3}b_1 + \frac{1}{2}b_1} = \frac{24}{13} \quad \boxed{A.}$$

30.) $(e^{i\theta})^3 = e^{3i\theta}$

$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta = \cos 3\theta + i\sin 3\theta$

Taking the real part of this...

$$\cos^3\theta - 3\cos\theta\sin^2\theta = \cos 3\theta$$

$$\cos^3\theta = \cos 3\theta + 3\cos\theta\sin^2\theta$$

$$\cos^3\theta = \cos 3\theta + 3\cos\theta - 3\cos^3\theta$$

$$\therefore \cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta \quad \boxed{C.}$$