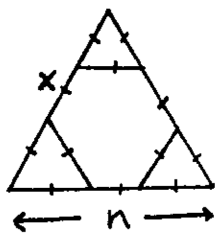
'93 PCHS INVITATIONAL GEOMETRY SOLUTIONS

INDIVIDUAL TEST

① 

$\frac{x^2\sqrt{3}}{4} \cdot 6 = \sqrt{3}$

$6x^2\sqrt{3} = 4\sqrt{3}$

$x^2 = \frac{2}{3}$, $x = \sqrt{\frac{2}{3}}$

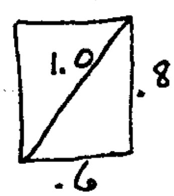
$x = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$n = \sqrt{6}$ $A = \frac{(\sqrt{6})^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$ $\boxed{C}$

② $2x + y = 2$

$x + 2y = 2.2$

$x = .6$ $y = .8$



$\boxed{D}$

③ $\boxed{A}$

④ $p = 240$, side $= 60$

$A = b \cdot h = 60 \cdot 30 = 1800$ $\boxed{E}$

⑤ the cycle will produce 8 diagonals. Total possible is $\frac{8 \cdot 5}{2} = 20$. $\frac{8}{20} = 40\%$ $\boxed{B}$

⑥ Find the base of the large Δ. $900 - 400 = 500$, base $= 10\sqrt{5}$. $x = 2\sqrt{5}$. Now find y. $400 + 20 = 420$. $y = \sqrt{420} = 2\sqrt{105}$

$xy = 2\sqrt{5} \cdot 2\sqrt{105} = 20\sqrt{21}$ $\boxed{C}$

⑦ $(x+2)^2 + (x-2)^2 = 400$

$2x^2 = 392$

$x^2 = 196$, $x = 14$. legs are 12 + 16.

$A = \frac{1}{2} \cdot 12 \cdot 16 = 96$ $\boxed{C}$

⑧ SA = 120, one face = 20

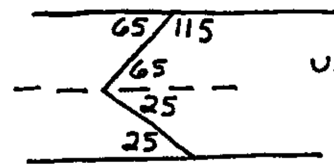
$s^2 = 20$, $s = 2\sqrt{5}$. $V = (2\sqrt{5})^3$

$V = 40\sqrt{5}$ $\boxed{C}$

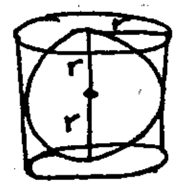
⑨ because of the right ∠,

$2y + 3y = 180$, $y = 36$.

$x = \frac{2(36) - 36}{2} = 18$ $\boxed{B}$

⑩ 

use alternate interior ∠'s. $\boxed{B}$

⑪ 

LA of cyl. $= 2\pi r \cdot 2r$

SA of sph. $= 4\pi r^2$

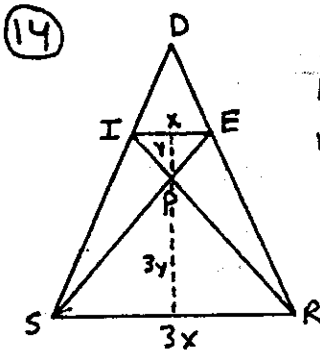
ratio is 1:1 $\boxed{A}$

⑫ $3\sqrt{6}$ = SL $6\sqrt{6}$ = Hyp. $9\sqrt{2}$ = LL $\boxed{B}$

⑬ $p = 14$ $s = 7$

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

$A = \sqrt{2 \cdot 3 \cdot 4 \cdot 5} = 2\sqrt{30}$ $\boxed{D}$

⑭ 

Area of Δ DIE = 10

Area of Δ SDR = 90

ratio of areas is 1:9

ratio of sides is 1:3

Area of SIER is $2x \cdot 4y$. $8xy = 80$

$xy = 10$

Area of Δ SPR $= \frac{9xy}{2} = \frac{9(10)}{2} = 45$ $\boxed{B}$

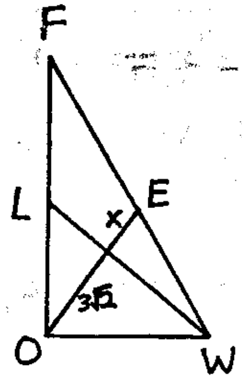
⑮ 

$\frac{x - (360 - x)}{2} = 22$

$2x - 360 = 44$

$x = 202$ $\boxed{E}$

(16)



$FE = EW = OE$

$2x = 3\sqrt{2}$ (medians trisect each other)

$x = \frac{3\sqrt{2}}{2}$

$FW = 2\left(3\sqrt{2} + \frac{3\sqrt{2}}{2}\right)$

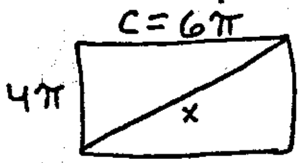
$FW = 6\sqrt{2} + 3\sqrt{2}$

$FW = 9\sqrt{2}$ $\boxed{C}$

(17) D (18) D

(19) opp. sides add to the same sum. $AM + GE = 9 + 8 = 17$ $\boxed{A}$

(20) unwrap the can.



$x = \sqrt{16\pi^2 + 36\pi^2}$

$x = \pi\sqrt{52}$

$x = 2\sqrt{13}\,\pi$ $\boxed{E}$

(21) Break the volume into 3 sections.

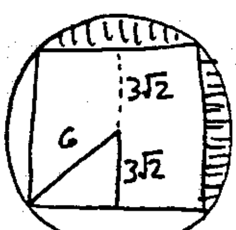
$V_1 = 4 \times 1 \times 5 = 20$

$V_2 = 4 \times 1 \times 5 + \frac{4 \times 4 \times 5}{2} = 60$

$V_3 = 4 \times 5 \times 5 = 100$. Total $= 180$ $\boxed{A}$

(22) $15x = 360$, $x = 24$, $y = \frac{72}{2} = 36$ $\boxed{A}$

(23)



$A = \frac{36\pi - 72}{2}$

$A = 18\pi - 36$ $\boxed{D}$

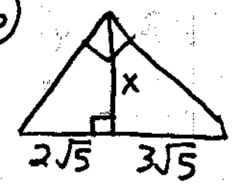
(24) The contrapositive

$\boxed{C}$

(25) $A = \frac{d_1 \cdot d_2}{2} = \frac{(12+\sqrt{2})(12-\sqrt{2})}{2}$.

$A = \frac{144-2}{2} = 71$ $\boxed{C}$

(26)



$x^2 = 2\sqrt{5} \cdot 3\sqrt{5}$

$x^2 = 30$

$x = \sqrt{30}$

$A = \frac{1}{2} \cdot 5\sqrt{5} \cdot \sqrt{30}$

$A = \frac{25\sqrt{6}}{2}$ $\boxed{E}$

(27) ${}_6C_3 = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} = 20$ $\boxed{D}$

(28) slope must be -1.

$y + 1 = -1(x + 1)$

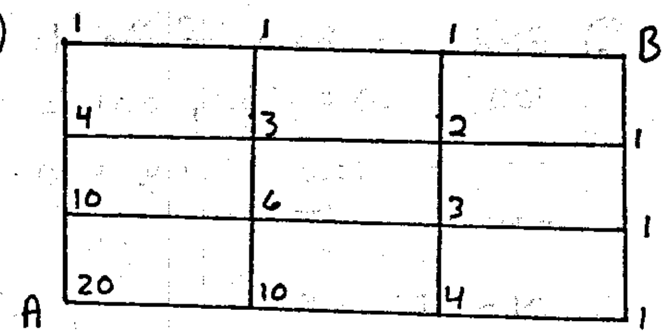
$x + y = -2$ $\boxed{D}$

(29) volumes (8 to 27)

sides ($\sqrt[3]{8}$ to $\sqrt[3]{27}$ or 2 to 3)

Areas (2^2 to 3^2 or 4:9) $\boxed{B}$

(30)



The number of ways to get to B from each point is given.

Another way: how many ways can you arrange this "word" UUURRR

$\frac{6!}{3!\,3!} = 20.$ $\boxed{E}$