

Calculus Team Question #1

Plant City High School

March 13, 1999

1. When the equation of the line tangent to the graph of  $f(x) = x \cos(4x)$  at  $x = \pi/4$  is written in the form  $y = mx + b$ , find  $2m - 2b$ .

Calculus Team Question #2

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2. Use the differential equation:

$$\frac{dy}{dx} = \sqrt{xy - 2x}$$

and the condition,  $y(1) = 3$ , to solve for  $y$  when  $x = 4$ .

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3. Approximate the value of the first quadrant area enclosed by  $y = 3 - x^2$  and  $y = -x + 1$  to the nearest hundredth.

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4. Let:

$$A = \lim_{x \rightarrow 1^-} \left( (x - 1) \tan \left( \frac{\pi x}{2} \right) \right)$$

$$B = \lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{\sin x^2}$$

$$C = \lim_{x \rightarrow \infty} \left( (8x^3 + 5x^2 + 1)^{1/3} - 2x \right)$$

$$D = \lim_{x \rightarrow -\infty} (2e^x + x^2)^{3/x}$$

Find:  $\sin(A^{-1}) + B + 24C + D$ .

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5. Let  $A$  be the average value of  $f(x) = 2 + |x|$  on the interval  $[-2, 2]$  and let  $B$  be the value for  $c$  described by the Intermediate Value Theorem so that  $f(c) = 4$  on the interval  $[-3, 1]$ . Evaluate:

$$\int_B^A f(x) dx$$

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6. The cross section of a 5-meter long trough is an equilateral triangle with a 9-meter side. Water is running into the trough at a rate of 1 cubic meter per second. How fast is the water level rising when the water is 1 meter deep? Answer in meters per minute.

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7. Evaluate:

$$\int_{-1}^1 \frac{dz}{\sqrt{z^2 + 2z + 2}}$$

Calculus Team Question #8

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8. What is the volume of the solid that results when the region bounded by  $y = -x + 6$ , the  $x$ -axis, and the  $y$ -axis, is rotated around the line  $4x - 3y - 19 = 0$ ?

Calculus Team Question #9

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9. The population of the United States was 205 million in 1970. Assuming an annual growth rate of 1.8%, find the year in which the population will reach 1 billion.

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10. Given that  $\int_1^5 P(x) dx = -1$ ,  $\int_2^5 P(x) dx = 3$ , and  $\int_2^5 Q(x) dx = 4$  let:

$$A = \int_2^5 (2P(x) + Q(x)) dx$$

$$B = \int_2^1 P(t) dt$$

$$C = \int_{-3}^{-5} Q(-x) dx$$

$$D = \int_1^2 P(x) dx$$

Find:  $A - 9B + C + D$

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11. An isosceles triangle is drawn with its vertex at the origin and its base above and parallel to the  $x$  axis. The vertices of the base are on the curve  $5y = 25 - x^2$ . Find the area of the normalsized such triangle.

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13. Find the value of  $\frac{d^2y}{dx^2}$  when  $x = 1$  given

$$y = \int_{x^2}^2 \ln(1 + \sqrt{t}) dt$$

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12. Suppose that  $f$  is continuous and  $x^2 \leq f(x) \leq 6$  for all  $x$  in  $[-1, 2]$ . Find values  $A$  and  $B$  such that:

$$A \leq \int_{-1}^2 f(x) dx \leq B$$

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14. Determine the values of  $k$  such that the function  $f(x) = kx + \sin x$  has an inverse.

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15. Evaluate:

$$\int_0^1 \sqrt{x} \ln x \, dx$$