#9

PLANT CITY HIGH SCHOOL
CALCULUS
MARCH 13, 1999

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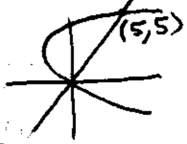
1. B
2. E (-1,0)
3. B
4. A
5. D
6. B
7. C
8. D
9. B
10. C
11. D
12. E 511/32
13. B
14. A
15. B
16. E 1
17. D
18. B
19. C
20. B
21. D
22. D
23. C
24. A
25. D
26. B
27. D
28. C
29. E 1/3
30. D

Calculus Ind. Solutions Plant City Invitational 1999

1. $\boxed{B}$



$$A = \int_0^5 y - (y^2 - 4y)\,dy = \frac{125}{6}$$

2. $\boxed{E}$ Minimize $x^2 + y^2$ where $y^2 = \frac{5}{2}(x+1)$

$$x^2 + \frac{5}{2}x + \frac{5}{2}$$

$$2x + \frac{5}{2} = 0$$

$$x = \frac{-5}{4}$$

$\Rightarrow$ not on curve

$\therefore$ $(-1,0)$ is closest

3. $\boxed{B}$ $\dfrac{(x-1)^{k+1}}{2k+3} \cdot \dfrac{2k+1}{(x-1)^k} \Rightarrow$

$|x-1| < 1 \Rightarrow (0,2)$

Check endpts. $0 \checkmark \displaystyle\sum_{x=1}^{\infty} \frac{(-1)^k}{2k+1}$

$[0,2)$ $\quad 2 \times \displaystyle\sum_{x=1}^{\infty} \frac{1}{2k+1}$

4. $\boxed{A}$ $\dfrac{f(b)-f(a)}{b-a} = f'(c)$

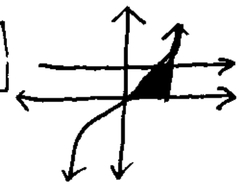
$$\frac{f(1)-f(0)}{1-0} = f'(c)$$

$$\frac{2-3}{1-0} = -2c$$

$$-1 = -2c$$

$$c = \frac{1}{2}$$

5. $\boxed{D}$



$$V = \pi \int_0^2 4^2 - \left(4 - \frac{x^3}{2}\right)^2 dx = \frac{80\pi}{7}$$

6. $\boxed{B}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2 - \frac{\sin 2 + \cos 2}{\cos 2 - \sin 2} \approx 2.372$$

$$x_3 = 2.372 - \frac{\sin 2.372 + \cos 2.372}{\cos 2.372 - \sin 2.372} \approx 2.356$$

7. $\boxed{C}$ $\displaystyle\int_0^1 \frac{1}{\sqrt{2+4x^2}}\,dx$

$$x = \frac{1}{\sqrt{2}} \tan\theta$$

$$dx = \frac{1}{\sqrt{2}} \sec^2\theta$$

$$\int_0^{\tan^{-1}\sqrt{2}} \frac{1}{\sqrt{2+2\tan^2\theta}} \cdot \frac{1}{\sqrt{2}} \sec^2\theta$$

$$\int_0^{\tan^{-1}\sqrt{2}} \frac{\sec^2\theta}{2\sec^2\theta}\,d\theta = \frac{1}{2}\int_0^{\tan^{-1}\sqrt{2}} \sec\theta\,d\theta$$

$$\frac{1}{2} \ln|\sec\theta + \tan\theta| \Big]_0^{\tan^{-1}\sqrt{2}}$$

$$\frac{1}{2} \ln|\sqrt{3} + \sqrt{2}|$$

8. $\boxed{D}$ $\quad V = \pi r^2 h$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r \frac{dr}{dt} h$$

$$\frac{dV}{dt} = \pi(6^2)(.4) + 2\pi(6)(.2)(3)$$

$$\frac{dV}{dt} = 21.6\pi$$

9. $\boxed{B}$

$4x^3 + 4y\,y' = 0$

$y' = \frac{-4x^3}{4y}$

$y' = \frac{-1}{\sqrt{3}}$

$\Rightarrow m = \sqrt{3}$

10. $\boxed{C}$

$v(t) = 3t^2 - 18t + 24 = 0$

$(t-2)(t-4) = 0$

$D = \left|\int_1^2 v(t)\,dt\right| + \left|\int_2^3 v(t)\,dt\right|$

$4 + 2 = 6$

11. $\boxed{D}$

$y' = \frac{x^3 - c^3 - (x+c^2)(3x^2)}{(x^3-c^3)^2}$

$y'(0) = \frac{-c^3}{c^6} = \frac{-1}{c^3}$

12. $\boxed{E}$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

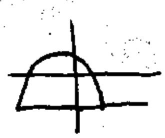
$x = 256$

$dx = 1$

$dy = \frac{1}{32}$

$y \approx 16 - \frac{1}{32} = \frac{511}{32}$

13. $\boxed{B}$



$y = \sqrt{9-(x+2)^2} - 2$

Forms a sphere of radius 3

$V = \frac{4}{3}\pi r^3 = 36\pi$

14. $\boxed{A}$

$f'(x)$	$2 \cdot 4^{2x+1} \cdot \ln 4$
$f^2(x)$	$4 \cdot 4^{2x+1} \cdot (\ln 4)^2$
$f^3(x)$	$8 \cdot 4^{2x+1} \cdot (\ln 4)^3$
$f^4(x)$	$16 \cdot 4^{2x+1} \cdot (\ln 4)^4$

$\Rightarrow 2^n \cdot 4^{2x+1} (\ln 4)^n$

$= 2^n \cdot 4^{2x+1} \cdot 2^n \cdot (\ln 2)^n$

$= 2^{4x+2n+1} \cdot (\ln 2)^n$

15. $\boxed{B}$

$P(x) = R(x) - C(x)$

$P(x) = 2x^2 + 5x - 400 - (\frac{1}{10}x^3 - 4x^2 + 5$

$= \frac{-1}{10}x^3 + 6x^2 - 406$

$\frac{dP}{dx} = \frac{-3}{10}x^2 + 12x$

$\frac{d^2P}{dx^2} = \frac{-3}{5}x + 12$

$x = 20$

16. $\boxed{E}$

$f'(x) = \frac{x}{(x^2+1)^4}$

$f''(x) = \frac{(x^2+1)^4 - x(8x)(x^2+1)^3}{(x^2+1)^8}$

at $x = 0 \Rightarrow f''(0) = 1$

17. $\boxed{D}$

$\frac{1}{b-a}\int_a^b 2x+5\,dx = \frac{1}{b-a}\; x^2+5x\Big|_a^b$

$\frac{b^2+5b-a^2-5a}{b-a} = \frac{(b-a)(b+a+5)}{b-a}$

$a+b+5$

18. $\boxed{B}$ $r = 2\left(\frac{y}{r}\right) - \left(\frac{x}{r}\right)$

$r^2 = 2y - x$

$x^2 + y^2 = 2y - x$

$x^2 + x + \frac{1}{4} + y^2 - 2y + 1 = \frac{5}{4}$

$\Rightarrow \frac{5}{4}\pi$

19. $\boxed{C}$ $P = \frac{k}{V}$ $k = 500$

$\int_1^2 \frac{500}{V}\,dV \approx 346.6$

20. $\boxed{B}$ $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y} y' = \ln x + 1$

$y' = x^x(1 + \ln x) = 0$

$\ln x = -1$

$x = e^{-1}$

21. $\boxed{D}$ $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$ $m_1 = f'(x)$ $m_2 = g'(x)$

$m_1 = 2 - 2(3) = -4$

$m_2 = 6 - 4 = 2$

$\theta \approx 40.6$

22. $\boxed{D}$ $a = a$

$v = at + V_0$

$s = \frac{1}{2}at^2 + V_0 t$

$300 = \frac{1}{2}a8^2 + V_0(8)$

$50 = 8a + V_0$

$a = 3.125$

$V_0 = 25$

23. $\boxed{C}$ $\lim_{\theta \to 0} \frac{\sin\theta - \tan\theta}{\sin^3\theta} = \lim_{\theta \to 0} \frac{1 - \frac{1}{\cos\theta}}{\sin^2\theta}$

$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\cos\theta(1 - \cos^2\theta)} = \frac{-\cos\theta + 1}{\cos\theta(1 + \cos\theta)(1 - \cos\theta)}$

$\lim_{\theta \to 0} \frac{-1}{\cos\theta(1 + \cos\theta)} = \frac{-1}{2}$

24. $\boxed{A}$ $y' = \frac{2\sqrt{x}}{1 + y}$

$m = \frac{1}{4}$

$\frac{\pi}{4} = \frac{1}{4} + b$

$b = \frac{\pi - 1}{4}$

$y = \frac{1}{4}x + \frac{\pi - 1}{4}$

$4y = x + \pi - 1$

$x - 4y + \pi - 1 = 0$

25. $\boxed{D}$ $\lim_{h \to 0} \frac{\sqrt{3 + h} - \sqrt{3}}{h} = f'(x)$

for $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

26. $\boxed{B}$ $\int_0^1 \int_0^{2x} e^{x^2}\,dy\,dx = \int_0^1 y e^{x^2}\Big]_0^{2x} dx$

$\int_0^1 2x e^{x^2}\,dx = e^{x^2}\Big]_0^1 = e - 1$

27. $\boxed{D}$ $y = \frac{\ln x}{x}$ $y' = \frac{1 - \ln x}{x^2}$

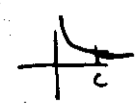
$y'' = \frac{x^2\left(\frac{-1}{x}\right) - 2x(1 - \ln x)}{x^4} = 0$

$-x - 2x + 2x\ln x = 0$

$x(-3 + 2\ln x) = 0$

$\ln x = \frac{3}{2}$

$x = e^{3/2}$

28. $\boxed{C}$ 

$\int_0^c f(x)\,dx = \frac{1}{2}\int_0^\infty f(x)\,dx$

$\int \frac{1}{(x+1)\sqrt{x}}\,dx = \int \frac{2u\,du}{(u^2+1)u} = 2\tan^{-1}u + C$

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$2\tan^{-1}u\Big]_0^{\sqrt{c}} = \frac{1}{2} \cdot 2\tan^{-1}u\Big]_{\sqrt{c}}^{\infty}$

$3\tan^{-1}\sqrt{c} = \pi/2$

$\sqrt{c} = \sqrt{3}$

$c = \frac{1}{\;}$

29. $x(t) = t^3 - 4$

$\boxed{E}$ $y(t) = t + 1$

$t = y - 1$

$x = (y-1)^3 - 4$

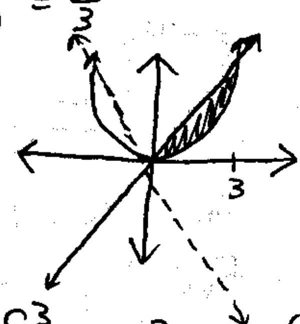
$1 = 3(y-1)^2 y'$

$y' = \dfrac{1}{3(y-1)^2}$

$t = 1 \rightarrow y = 2$

$y' = \frac{1}{3}$

30. $\boxed{D}$



$\text{Area} = \int_0^3 3x - x^2\,dx = \dfrac{9}{2}$

Centroid:

$\bar{x} = \dfrac{\int_0^3 x(3x - x^2)\,dx}{\int_0^3 3x - x^2\,dx} = \dfrac{3}{2}$

$\bar{y} = \dfrac{\frac{1}{2}\int_0^3 \left((3x)^2 - (x^2)^2\right)dx}{\int_0^3 3x - x^2\,dx} = \dfrac{18}{5}$

$\left(\dfrac{3}{2}, \dfrac{18}{5}\right)$

$V = 2\pi\,(\text{Area})\left(\begin{array}{c}\text{Distance centroid}\\ \text{is from line of rotation}\end{array}\right)$

$V = 2\pi\left(\dfrac{9}{2}\right)\left(\dfrac{12\left(\frac{3}{2}\right) + 5\left(\frac{18}{5}\right) + 3}{\sqrt{12^2 + 5^2}}\right)$

$V = 9\pi \cdot \left(\dfrac{18 + 18 + 3}{13}\right)$

$V = 27\pi$

Team Round

1. $f(x) = x\cos 4x$

$f'(x) = \cos 4x - 4x\sin 4x$

$m = -1$

$y = -x + b$

$\frac{-\pi}{4} = \frac{-\pi}{4} + b$

$b = 0$

$2m - 2b = \boxed{-2}$

2. $\dfrac{dy}{dx} = \sqrt{x}\sqrt{y-2}$

$\int \dfrac{dy}{\sqrt{y-2}} = \int \sqrt{x}\,dx$

$2(y-2)^{1/2} = \frac{2}{3}x^{3/2} + C$

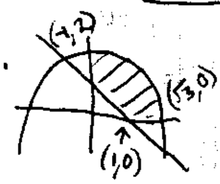
$2(3-2)^{1/2} = \frac{2}{3}(1)^{3/2} + C$

$C = \frac{4}{3}$

$2(y-2)^{1/2} = \frac{2}{3}(4)^{\frac{3}{2}} + \frac{4}{3}$

$\boxed{y = \dfrac{118}{9}}$

3.



$A = \int_0^{\sqrt{3}} 3 - x^2\,dx - \int_0^1 -x + 1\,dx$

$A = 3x - \frac{x^3}{3}\Big|_0^{\sqrt{3}} - \left(\frac{-x^2}{2} + x\Big|_0^1\right)$

$A = 2\sqrt{3} - \frac{1}{2}$

$\boxed{A \approx 2.96}$