PLANT CITY HIGH SCHOOL INVITATIONAL - FEBRUARY 6, 1993 - CALCULUS INDIVIDUAL

1. A stone is thrown directly upward from the ground with a velocity of 96 ft/sec. How far does the stone travel until one second before it hits the ground ?

   a. 196 ft. b. 208 ft. c. 220 ft. d. 232 ft. e. NOTA

2. Find $D_x y$ if $y = 2^{x^2}$.

   a. $2x^2 \ln(x)$ b. $x 2^{x^2+1} \ln(2)$ c. $x^2 2^{x^2-1}$ d. $2^{x^2} x^2 \ln(2)$

   e. NOTA

3. Two cars, X and Y, 120 miles apart, travel towards each other at rates of x and y miles per hour respectively. If both start at 12:00 noon, they will meet at 1:30 pm. If X starts at 11:30 am and Y starts at 12:30 pm, they will also meet at 1:30 pm. Find the value of x-y.

   a. 40 b. -40 c. 20 d. 0 e. NOTA

4. Let $f(x) = [\![x]\!]$ denote the greatest integer function. Find

   $$\lim_{x \to -4^-} x + 1 - [\![x]\!]$$

   a. -1 b. 0 c. 1 d. 2 e. NOTA

5. Use the trapezoidal rule to approximate $\int_0^1 4x^3 \, dx$ with n=4.

   a. 1 b. $\frac{17}{16}$ c. $\frac{9}{8}$ d. $\frac{19}{16}$ e. NOTA

6. Suppose the substitution $z = \tan(\frac{x}{2})$ is appropriate. In terms of z, $\cos(x) = \ldots$

a. $(1+z^2)/(1-z^2)$ b. $(2z)/(1+z^2)$ c. $(2z)/(1-z^2)$

d. $(1-z^2)/(1+z^2)$ e. NOTA

7. If u and v are each differentiable functions of x for a given interval, then $\frac{d}{dx}(\frac{u^n}{v^m}) =$

a. $\frac{nu^{n-1}\ du/dx}{mv^{m-1}\ dv/dx}$ b. $\frac{u^{n-1}}{v^{m+1}}(nv\frac{dv}{dx} - mu\frac{du}{dx})$

c. $\frac{u^{n-1}}{v^{2m}}(nv\frac{du}{dx} - mu\frac{dv}{dx})$ d. $\frac{u^{n-1}}{v^{m+1}}(nv\frac{du}{dx} - mu\frac{dv}{dx})$ e. NOTA

8. A force of 20 lbs. is needed to stretch a spring from its natural length of 14 inches to a length of 18 inches. Find the work in lbs-in needed to stretch the spring from 18 inches to 24 inches.

a. 230 b. 120 c. 180 d. 210 e. NOTA

9. Find the average value of $f(x) = \sec(x)$ with respects to x from $x = \pi/4$ to $x = \pi/3$.

a. $\frac{12}{\pi}\ln(\sqrt{6} - \sqrt{3} + 2\sqrt{2} - 2)$ b. $\frac{12}{\pi}\ln(\sqrt{6} + \sqrt{3} - 2\sqrt{2} + 2)$

c. $\frac{12}{\pi}\ln(\sqrt{6} - \sqrt{3} - 2\sqrt{2} + 2)$ d. $\frac{12}{\pi}\ln(\sqrt{6} + \sqrt{3} + 2\sqrt{2} + 2)$

e. NOTA

10. If $f(x) = 2x^3-3x^2-36x+7$, find the absolute minimum of its derivative on $[-3,4]$.

a. 51 b. $-\frac{23}{2}$ c. $-\frac{75}{2}$ d. $\frac{1}{2}$ e. NOTA

11. At any point (x,y) on a curve, $y''' = -6$ and (1,5) is a point of inflection at which the slope of the inflectional tangent is 2. For the point (3,y) on the curve, find y.

a. $\frac{3}{2}$ b. 2 c. $\frac{5}{2}$ d. 3 e. NOTA

12. The position of a particle P(x,y) at any time t is given by

$x = \frac{1}{3}(2t+3)^{3/2}$, $y = \frac{t^2}{2} + t$. Find the distance the particle travels from t=0 to t=3.

a. $\frac{21}{2}$ b. $\frac{3}{2}$ c. $\frac{15}{2}$ d. 10 e. NOTA

13. Given two concentric circles, C_1 of radius r_1 and C_2 of radius r_2, $r_2>r_1>0$. Let A be the area between the two circles. Suppose at time t=0, r_1 is 3 cm and r_2 is 5 cm, and that for t>0, r_1 increases at the constant rate of a cm/sec and r_2 increases at the constant rate of b cm/sec. If $\frac{3}{5}a<b<a$, find time t when the area A will be a maximum.

a. $\frac{3b-5a}{a^2-b^2}$ b. $\frac{3b-5a}{b^2-a^2}$ c. $\frac{5b-3a}{a^2-b^2}$ d. $\frac{5b-3a}{b^2-a^2}$ e. NOTA

14. Find $\frac{dy}{dx}$ given $y = x^2e^x\cos(x)$.

a. $y(1 - \frac{2}{x} + \tan(x))$ b. $y(1 + \frac{2}{x} - \cot(x))$

c. $y(1 + \frac{2}{x} - \tan(x))$ d. $y(1 - \frac{2}{x} + \cot(x))$ e. NOTA

15. $\frac{d}{dx}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\right) =$

a. $\sin(x)$ b. $\cos(x)$ c. e^x d. $\ln(x)$ e. NOTA

16. If $y=f(u)$ and $u=g(x)$ and y and u are differentiable functions of u and x respectively, then the derivative of y with respects to x is ...

a. $f'(g(x))g'(x)$ b. $f(g'(x))f'(x)$ c. $f'(g(x))f'(x)$

d. $f(g'(x))g'(x)$ e. NOTA

17. The curve $y = \frac{x^2}{2} + \frac{1}{2}$ is rotated about the y-axis. Find the area of the surface generated from x=0 to x=1.

a. $\frac{2\pi(2\sqrt{2}-1)}{3}$ b. $\frac{2\pi(2\sqrt{2}+1)}{3}$ c. $\frac{2\pi(3\sqrt{2}-1)}{3}$ d. $\frac{2\pi(3\sqrt{2}+1)}{3}$

e. NOTA

18. Let f be an even function with domain defined for all real numbers. Which of the following statements must be true?

a. $f'(x) = f'(-x)$ b. $f'(x) = -f'(-x)$ c. $f'(x) > f'(-x)$

d. $f'(x) < f'(-x)$ e. NOTA

19. He has been called the Japanese Newton, probably discovering the calculus of the East just as Newton discovered the calculus of the West. What is his name?

a. Yoko Mōhata b. Seki Kōwa c. Yanku Kōshuma

d. Suyo Mōkita e. NOTA

20. $\lim_{x\to\infty} \frac{4x - 1/(2x)}{5x + 1/(4x)} =$

a. 0 b. 1 c. $\frac{4}{5}$ d. ∞ e. NOTA

21. What percentage rate of interest, compounded continuously for one hundred years, would multiply an original capital investment A_o by 90?

a. $\ln(A_o)$ b. $\ln(100)$ c. $\ln(90)$ d. $100\ln(A_o)$ e. NOTA

22. $\int_{\pi/6}^{\pi/2} \cos(x)\ln(\sin(x))\,dx =$

a. $\frac{1}{2} + \frac{1}{2}\ln 2$ b. $-\frac{1}{2} + \frac{1}{2}\ln 2$ c. $\frac{1}{2} - \frac{1}{2}\ln 2$

d. $2 - \frac{1}{2}\ln 2$ e. NOTA

23. $\lim_{x\to\infty} x^{1/x}$

a. ∞ b. 1 c. 0 d. Does not exist e. NOTA

24. Find $\frac{dy}{dx}$ if $y = \arcsin(3x)$.

a. $3(1+9x^2)^{-1/2}$ b. $(1+9x^2)^{-1/2}$ c. $3(1+3x^2)^{-1/2}$

d. $3(1-3x^2)^{-1/2}$ e. NOTA

25. If $f(a)=f(b)=0$, and $f(x)$ is continuous on $[a,b]$, then ...

a. $f(x)$ must be identically zero.
b. there must exist at least one number c, $a<c<b$, such that $f'(c)=0$.
c. $f'(x)$ may be different from zero for all x on $[a,b]$.
d. $f'(x)$ may be negative for all x on $[a,b]$.

26. If $R = 3\cos(\frac{\pi}{3}t)i + 2\sin(\frac{\pi}{3}t)j$ is the position vector $xi+yj$ from the origin to a moving point $P(x,y)$ at time t, write the equation of the position in terms of x and y only.

a. $4x^2+9y^2=36$ b. $9x^2+4y^2=36$ c. $4x^2+9y^2=1$ d. $9x^2+4y^2+1$ e. NOTA

27. $\frac{d}{dx}\int_{\cos(x)}^{0} \frac{dt}{2+t} =$

a. $-\ln|2+\cos(x)| + 1$ b. $\ln|2+\cos(x)| - 1$

c. $\frac{\cos(x)}{2+\sin(x)}$ d. $\frac{-\sin(x)}{2+\cos(x)}$ e. NOTA

28. A ball 60 cm in diameter is floating so that the top of the ball is 12 cm above the smooth surface of the water. Find the circumference of the circle formed by the contact of the water surface with the ball.

a. 48π b. 36π c. 24π d. 18π e. NOTA

29. The cross sections of a certain solid by planes perpendicular to the x-axis are circles with diameters extending from the curve $y=x^2$ to the curve $y=8-x^2$. The solid lies between the points of intersection of these two curves. Find its volume.

a. $\frac{512\pi}{15}$ b. $\frac{256\pi}{15}$ c. $\frac{128\pi}{15}$ d. $\frac{64\pi}{15}$ e. NOTA

30. $\int \frac{(2x+3)\,dx}{4x^2+4x+5} =$

a. $\frac{1}{4}\ln|4x^2+4x+5| + \frac{1}{2}\arctan(2x+3) + C$

b. $\frac{1}{4}\ln|4x^2+4x+5| + \frac{1}{2}\arctan(\frac{2x+1}{2}) + C$

c. $\frac{1}{4}\ln|4x^2+4x+5| + \frac{1}{2}\arctan(\frac{2x+3}{2}) + C$

d. $\frac{1}{4}\ln|4x^2+4x+5| + \frac{1}{2}\arctan(2x+1) + C$

e. NOTA