

Plant City High School
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MAΘ Invitational
Algebra II Team

Solutions

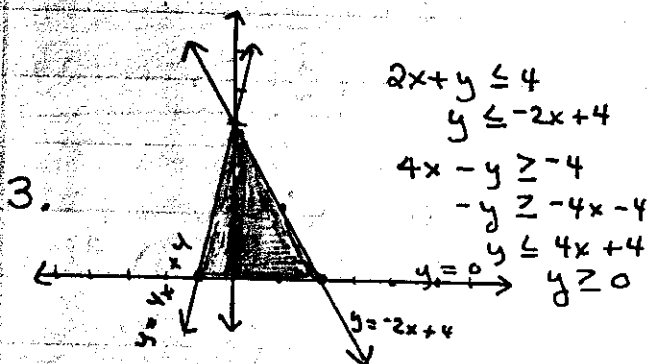
1. $A = lw$

$$a^2 - b^2 = \frac{a-b}{a^2+ab} w$$

$$(a-b)(a+b) \cdot \frac{a(a+b)}{a-b} = w$$

$$w = a(a+b)^2$$

$$w = a^3 + 2a^2b + ab^2$$



$$2x + y \leq 4$$

$$y \leq -2x + 4$$

$$4x - y \geq -4$$

$$-y \geq -4x - 4$$

$$y \leq 4x + 4$$

$$y \geq 0$$

3.

$$A = \frac{1}{2}hb$$

$$A = \frac{1}{2}(4)(3)$$

$$A = 6 \text{ sq units}$$

5. $A: \frac{(x-1)^2}{1/4} + \frac{(y+1)^2}{1/9} = 1$
If $a^2 = 1/4$ then $a = 1/2$
major axis is $2(\frac{1}{2}) = 1$

$$B: \sqrt{(1-2)^2 + (4-3)^2}$$

$$\sqrt{1+49} = \sqrt{50}$$

$$C: t_{25} = 8 + (25-1)(1.5)$$

$$t_{25} = -28$$

Find $2A - C + B^2$

$$2(1) - (-28) + (\sqrt{50})^2$$

$$2 + 28 + 50$$

(80)

2. t - ten's digit
 u - one's digit

$$10t + u = 4(t+u)$$

$$10u + t = 2(10t+u) - 6$$

$$\begin{cases} 6t - 3u = 0 & \times \frac{8}{3} \\ -19t + 8u = -6 \end{cases} \quad \begin{cases} 12-3u=0 \\ -3u=-12 \\ u=4 \end{cases}$$

$$-3t = -6$$

$$t = 2$$

(24)

4. $\sqrt{2x-5} = \sqrt[3]{2x-5}$

$$(2x-5)^3 = (2x-5)^2$$

$$(2x-5)^3 - (2x-5)^2 = 0$$

$$(2x-5)^2(2x-5-1) = 0$$

$$\begin{matrix} 2x-5=0 & 2x-6=0 \\ 2x=5 & 2x=6 \\ x=5/2 & x=3 \end{matrix}$$

{5/2, 3}

6. $-3 = \frac{4+a}{2}$

$$-6 = 4+a$$

$$-10 = a$$

$$5 = \frac{b+3}{2}$$

$$10 = b+3$$

$$7 = b$$

$$a+b = -10+7 = -3$$

7. $m = \frac{1-7}{2-6}$
 $m = \frac{8}{-4} = -2$
 $\perp m = 1/2$
 midpt = $(\frac{2+6}{2}, \frac{1+7}{2})$
 midpt = $(4, -3)$
 $y = mx + b$
 $-3 = \frac{1}{2}(4) + b$
 $-5 = b$
 $y = \frac{1}{2}x - 5$

8. $A = \text{Integers} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
 $B = 1 - x \geq 0$
 $-x \geq -1$
 $x \leq 1$
 $C = \begin{array}{c} \uparrow \\ \downarrow \\ \leftarrow \quad \rightarrow \\ \uparrow \quad \downarrow \end{array} \quad y \geq -2$
 $(A \cap B) \cap C$
 $\{1, 0, -1, -2, \dots\} \cap y \geq -2$
 $\{1, 0, -1, -2\}$

9. $\uparrow D = 36t$
 $\downarrow D = 14t$
 $r + c = 36$
 $\frac{1}{4}r - c = 14$
 $\frac{5}{4}r = 50$
 $r = 50(\frac{4}{5})$
 $r = 40 \text{ mph}$

10. $x^2 - 2x + y^2 + 8y = -12$
 $(x^2 - 2x + 1) + (y^2 + 8y + 16) = -12 + 1 + 16$
 $(x-1)^2 + (y+4)^2 = 5$
 Center $(1, -4)$
 $x = y^2 + 4y + 3$
 vertex $(4 - 8 + 3, \frac{-4}{2(1)})$
 vertex $(-1, -2)$
 Focus $(-1 + \frac{1}{4}, -2) = (\frac{-3}{4}, -2)$
 $D = \sqrt{(1 - \frac{3}{4})^2 + (-4 - (-2))^2}$
 $D = \sqrt{\frac{49}{16} + \frac{64}{16}} = \sqrt{\frac{113}{4}}$

11. $\frac{2x - 38}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4}$

I. $\frac{2x - 38}{x+2} = \frac{A(x-4)}{x+2} + B$

Let $x = 4$
 $\frac{2(4) - 38}{4 + 2} = 0 + B$
 $\frac{-30}{6} = B$
 $-5 = B$

II. $\frac{2x - 38}{x-4} = A + \frac{B(x+2)}{x-4}$

Let $x = -2$
 $\frac{2(-2) - 38}{-2 - 4} = A + 0$
 $\frac{-42}{-6} = A$
 $7 = A$

$A - B$
 $7 - (-5) = 12$

Characteristic

12. $A = 2$ $\log 347.6 = \log 3.476 \times 10^{\textcircled{2}} = \textcircled{2}.5410$

$B = 72$ $b^2 - 4ac = (-4)^2 - 4(2)(-7) = 72$

$C = 4/3$ $2 \log_8 4 = C$

$\frac{B}{A} = C$

$\frac{72}{2} = \frac{4}{3} = \textcircled{34.\bar{6}}$

$8^C = 16$

$2^{3C} = 2^4$

$3C = 4$
 $C = 4/3$

13. $t_4 = 8\frac{1}{2} = t_1$
 $t_8 = 16 = t_5$
 $t_{22} = ? = t_{19}$

$t_{22} = 42.25$

$16 = 8\frac{1}{2} + (5-1)d$

$7.5 = 4d$

$1.875 = d$

$t_{19} = 8\frac{1}{2} + (19-1)(1.875)$

$= 8.5 + (18)(1.875)$

$= 8.5 + 33.75$

$= 42.25$

14. $x^3 - 7x + 6 = 0$
roots $\{-3, 1, 2\}$

$(x+3)(x-1)(x-2) = 0$

sum = 0

product = -6

sum of the reciprocals of the roots

$-\frac{1}{3} + \frac{1}{1} + \frac{1}{2} = \frac{-2+6+3}{6} = \frac{7}{6}$

$0 + -6 - 7/6 = \textcircled{-7.\bar{16}}$

15. $8^{4x-5} = 4^{x+3} + 4^{x+3}$
 $2^{3(4x-5)} = 2(2^{2(x+3)})$
 $2^{12x-15} = 2^{2x+6+1}$

$12x - 15 = 2x + 7$

$10x = 22$

$x = 2.2$