2005 Palm Harbor February Invitational
Statistics Answer Key

Individual

1. C
2. D
3. C
4. C
5. A
6. C
7. C
8. D
9. C
10. A
11. C
12. B
13. D
14. B
15. A
16. A
17. B
18. D
19. E
20. B
21. A
22. A
23. E
24. D
25. D
26. A
27. B
28. A
29. D
30. B

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1) There are 10 combinations for getting three tails, 5 for getting four tails, and one for getting five tails. This makes a total of 16 combinations. The probability of any one combo is .5 to the fifth or 1/32. Therefore,

16 combos X (1/32) = 1/2 **C**

2) Only I, IV, V, and VI are non-resistant to outliers. Therefore, **D**

3) Since the area under the curve must be one, the height of this rectangle is found to be 1/ .6 or 5/3. The probability that a point is between .2 and .4 is therefore,

Probability = 5/3 X 1/5 (which is .4 - .2) = 1/3 **C**

4) If the regression line has the form ŷ = a + bX, then a = $\overline{Y} - b\overline{X}$.

Since $b = r\frac{s_y}{s_x} = .75 \times \frac{2}{3} = .5$ Therefore,

7 = 10 - .5 $\overline{X}$ and $\overline{X}$ = 6 **C**

5) If the mean is less than the median, then the data is skewed left because the mean is not resistant skew ness. Hence, **A**

6) Class Average: 86 = (81 + 95 + Twenty Other Scores) / 22 (# of students in class)
1892 = Twenty Other Scores + 176 -----→ Twenty Other Scores = 1716
The Average of Remaining Students is 1716 / 20 = 86% **C**

7) The probability of A and B given A = (.2)(.4) / (.2) = .4 **C**

8) Because the same sample is being compared before and after, it is a matched pairs design. Also since the population standard deviation is unknown, a t – test must be used. Therefore, a matched pairs t - test is best. **D**

9) Find the Z score that is closest to .90 in Z table, which is 1.28. Then solve:
1.28 = (X – 550) / 10 -----→ X = 563 **C**

10) Statements I, II, and III are false
I. Only modified box plots reveal outliers
II. The standard deviation is the square root of the variance
III. The mean is more left than the median
Statement IV is true.
IV. The mean and median are always equal in a normal distribution. And two standard deviations below the mean will exclude 2.5%

One statement is true. **A**

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11) The probability of X and Y = (Probability of X given Y) (Probability of Y)
= (.210) (.650) = .1365

Hence, the probability of Y given X = (Probability of X and Y) / (Probability of X)
= (.1365) / (.260) = .525 **C**

12) For a t test, degrees of freedom is n – 1, the P(type II error) is 1 – power of test, and the degrees of freedom for a regression test is n – 2.

Therefore, (22 – 1) (1 - .64) (13 – 2) = 83.16 ---→ Units place is 3. **B**

13) Draw a line through the two most extreme points and a straight line is formed with the variation of the outside two points being very insignificant. Scatter plot virtually resembles a line. Therefore, r approximately equals .99. **D**

14) Null hypothesis always represents no change, so the null is that the mean equals 70. Since John believes that mean is higher, the alternate is greater than 70. **B**

15) T test statistic = (sample mean – null hypothesis) / $(s / n^{1/2})$
= $(71.5 - 70) / (2/30^{1/2}) = 4.1079$

Since the test statistic has a probability of less than a .0002 and since it is lower than the alpha level of .01, there is sufficient evidence to reject the null in favor of the alternate. **A**

16) P(type I error) = alpha level = .01. Therefore, **A**

17) P(Winning the lottery) = $1 / [(56)(55)(54)(53)(52)(51) / 6!] = 3.07 \times 10^{-8}$
Since the lottery is played weekly, determine how many weeks there are in one million years:
52 X 1,000,000 = 52,000,000 weeks. Now for more than a million years, use the formula: $P(x > n) = (1 - p)^n =$
$P(x > 1{,}000{,}000) = (1 - 3.07 \times 10^{-8})^{52.00.000}$
P (X > 1,000,000) = .201582… Therefore, **B**

18) $z^* [p^* (1 - p^*) / n]^{1/2} \leq .02$ -----→ $1.960 [.5 (1 - .5) / n]^{1/2} \leq .02$
$1.960 [.5 \times .5]^{1/2} / (.02) \leq n^{1/2}$
$49 \leq n^{1/2}$
$n \geq 2401$ --→ n = 2401. Hence, **D**

19) Z score that corresponds to 25% greater than mean is .67. Therefore,
.67 = (3.0 – 2.7) / SD -------→ SD = .45 m. Hence, **E**

20) Chi squares are always skewed right not left. Therefore, **B**

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21) Distribution appears as follows:

X (# of Odds)	0	1	2	3
P(X)	$(.5)^3 = .125$	$(.5)^3$ X 3 = .375	$(.5)^3$ X 3 = .375	$(.5)^3 = .125$

Mean = 0 X .125 + 1 X 375 + 2 X .375 + 3 X .125 = 1.5
Variance = $(0 – 1.75)^2$ X .125 + $(1- 1.75)^2$ X .375 + $(2 – 1.75)^2$ + $(3 – 1.75)^3$ = .75
Therefore, **A**

22) The mean time to make 1 widget is 3 + 5 + 7 = 15 minutes, the mean time to make 9 widgets is 9*15 = 135 minutes. Standard deviation of time to make 1 widget is $\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$, standard deviation of average time to make 9 widgets is $\frac{\sqrt{3}}{\sqrt{9}} = \frac{1}{\sqrt{3}}$, so standard deviation of total time to make 9 widgets is $\frac{9}{\sqrt{3}} = 3\sqrt{3}$

$z = \frac{150 \quad 135}{3\sqrt{3}} = 2.887$ z-test: 0.001945 = 0.0019 **A**

23) P(A and B) = P(A) X P(B), P(Complement of B) = 1 – P(B), P(A or B) = P(A) + P(B) – P(A and B). Hence,

.314 X .271 - (1 - .271) - (.314 + .271 - .314 X .271) = -1.14. Hence, **E**

24) Geometric distributions do not have a finite number of trials. Therefore, **D**

25) Confidence interval = p hat $\pm z^* [p^* (1 – p^*) / n]^{1/2}$
$915/2250 \pm 1.645 [.5 \text{ X } .5 / 2250]^{1/2}$
$0.41 \pm .017$. Therefore, **D**

26) Mean of new distribution: 1 X 12 = 12 (Because one must add up the means of each ball to get the new mean of 12 balls)
SD: $(.1^2 \text{ X } 12)^{1/2} = .346$ (Because one must add the variances of each ball and then square root to get the SD of new distribution). Now standardize:
(13 – 12) / .346 = 2.88 ----→ Area greater than this is .0020. Hence **A**

27) This is response bias. **B**

28) Qualitative means no number value and ethnic background is just a categorical variable and not a numerical value. Therefore, **A**

29) A one tail test is most appropriate for a comparison of two things In one direction only, such as better or worse. Choices A, B, and C all compare two things in two directions and not just one. Therefore, D is the only appropriate choice. **D**

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30) Change to variance: $3.5^2 = 12.25 = \sigma^2$
$4.2^2 = 17.04 = \sigma^2$

$$\sigma^2{}_{x+y} = \sigma^2{}_x + \sigma^2{}_y + 2(p)\sigma_x\sigma_y$$
$$= 12.25 + 17.64 + 2(.7)(3.5)(4.2) = 50.47 = \sigma^2$$

$\sigma = 7.1042$ **B**