## 2004 Palm Harbor Invitational Pre-Calculus Team Question 1

Given: $f(\theta) = -\sin(\theta - 54°)$
$g(\theta) = 6 - 4\cos(16\theta)$
$h(\theta) = 2\csc(\theta) - 8.5$

A = The fewest number of degrees the graph of $f(\theta)$ must be shifted horizontally to the right in order to be identical to $y = \cos(\theta + 21°)$.
B = The number of degrees in the period of $g(\theta)$.
C = The sum of the vertical shifts of $f(\theta)$, $g(\theta)$, and $h(\theta)$.
D = The sum of the amplitudes of $f(\theta)$ and $g(\theta)$.

Find: A + B + C + D

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 2

Given: $M = \begin{bmatrix} 1 & 2 & -3 \\ -4 & 5 & -6 \end{bmatrix}$ $N = \begin{bmatrix} 6 & -5 \\ -3 & 4 \\ 2 & 1 \end{bmatrix}$ $O = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$

$\begin{vmatrix} -k & k & k \\ -k & k & -k \\ k & k & -k \end{vmatrix} = 500$ $P = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $Q = \begin{bmatrix} -4 & 1 \\ -3 & -2 \end{bmatrix}$

Find the sum of the elements in the matrix resulting from: $k \times M \times N \times O^{-1} + P - Q$

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 3

Where $i = \sqrt{-1}$,

A = $i^{2004}$

B = $(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)^{2004}$

C = The sixth root of $-1$ in the interval $(90°, 180°]$.

D = $\frac{32 + 56i}{2 + 2i}$

Find in $a + bi$ form: A + B + C + D

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 4

Find the **exact** value of the sum of the solutions (in degrees) to the following two trigonometric equations in the interval $[0°, 360°)$.

$$-\cot^2\theta = \cot\theta \qquad\qquad 2\cos 3\theta\cos 21° - 2\sin 3\theta\sin 21° = \sqrt{3}$$

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 5

Given: $f(x) = 2x + 8$

$g(x) = 2x^2 - 4$

$h(x) = \sqrt[3]{x-2}$

$$p(x) = h^{-1}(f(x)) - f^{-1}(g(x)) - h(h^{-1}(x)) + f(g(h(10)))$$

Find $p(2)$.

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 6

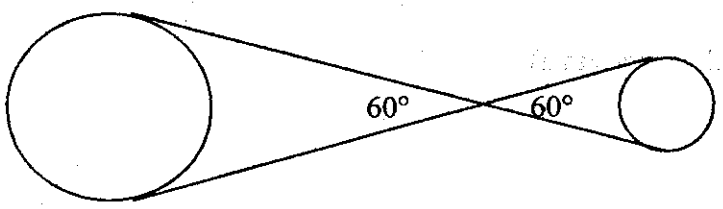
A = The number of distinct permutations of the singular noun commonly used for the type of conic represented by the equation $6x^2 - 12xy + 6y^2 + 2x - 3y - 144 = 0$.

B = The probability that a randomly chosen angle in the interval $(\frac{\pi}{3}, \frac{5\pi}{3})$ has a positive cotangent but negative secant.

Find: $A \times B$ (exact value)

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 7

Two circular gears, one 8 cm and the other 4 cm in radius, are connected by a belt around a portion of their outer edges such that the belt crosses itself between them (see diagram below). If the angle at which the belt crosses itself is $60°$, what is the **exact** total length of the belt? (Hint: The belt departs from the edge of the gear tangent to the gear.)



## 2004 Palm Harbor Invitational Pre-Calculus Team Question 8

$A = \lim_{x\to\infty} \dfrac{(x+1)(x+4)(x+3)}{(x-3)(3x-8)(4-x)}$

$B = \lim_{x\to\pi} \cos[2(x-\dfrac{\pi}{6})]$

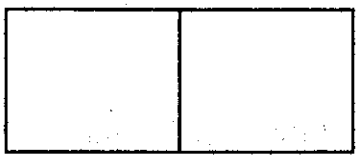
$C = \lim_{x\to 3} \dfrac{x^2+4x-21}{x^2-x-6}$

$D = \lim_{x\to-\infty} \dfrac{6x^2-2x-3}{x^2-60x-120}$

Find: $A \times B \times C \times D$ (exact value)

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 9

Mr. Macfarlane has 180 yards of fencing available. If he is to build a fence that encloses two rectangular corrals of identical shape and size such that they are adjacent to each other and share one side (see diagram below), what is the maximum area that his fencing can enclose in square yards?



## 2004 Palm Harbor Invitational Pre-Calculus Team Question 10

If $f(x)$ is a parabola with focus at (2,3) and directrix y = 5, then what is $f(2)$?

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 11

Given: $f(x) = \dfrac{x^2+x-30}{-x^2+7x-10}$

$g(x) = 32x^6 + 24x^3 - 48x^2 + 3x + 8$

$h(x) = 1 + 2\tan(2x - \dfrac{\pi}{2})$

A = $m+n$, where $x = m$ and $y = n$ are the equations for the asymptotes of $f(x)$.

B = The remainder when $g(x)$ is divided by $2x+3$.

C = The sum of the $x$ values (in radians) at which $h(x)$ is not continuous in the interval $[0, 2\pi]$.

Find: A + B + C (exact value)

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 12

Given: $g(x) = \dfrac{2x^2 - 8}{x - 2}$

$h(x) = \sqrt{36 - x^2}$

How many integers are in the range of the composite function given by $g(h(x))$?

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 13

Given:

I. $x^2 - 4y^2 - 4x - 24y - 36 = 0$

II. $x^2 + y^2 + 8x - 10y - 40 = 0$

A = The distance between the two foci of conic I.
B = The volume of a sphere with the radius of conic II.
C = The eccentricity of conic II.

Find: A + B + C (exact value)

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 14

Given:

Vectors

$\vec{a} = 2\vec{i} + 5\vec{j} + 3\vec{k}$

$\vec{b} = 7\vec{i} - \vec{j} + 4\vec{k}$

$\vec{w} = 2\vec{i} + 3\vec{j} + 6\vec{k}$

$\vec{v} = 3\vec{i} - 4\vec{j} + 12\vec{k}$

Planes

I. $3x + 2y - 5z + 11 = 0$

II. $4x + 7y + z - 8 = 0$

A = the angle between $\vec{a}$ and $\vec{b}$ (in radians)
B = the smaller dihedral angle between planes I and II (in radians)
C = the area of a parallelogram that is determined by $\vec{w}$ and $\vec{v}$ as two of its sides

Find: A + B + C (to the nearest hundredth)

## 2004 Palm Harbor Invitational Pre-Calculus Team Question 15

David stands on the ground watching an airplane pass directly overhead. When he first looks at the airplane, its angle of elevation from him is $30°$; one minute later its angle of elevation from him is $60°$ and it has already passed over him. If the plane is flying 1000 meters high and travels at a constant velocity, how far away from David (to the nearest hundredth of a meter) will the plane be two minutes after David first began watching it?