2005 Palm Harbor February Invitational
Pre-Calculus Answer Key

Individual

1. D
2. D
3. E
4. C
5. D
6. C
7. A
8. A
9. E
10. B
11. E
12. B
13. D
14. A
15. A
16. C
17. B
18. B
19. E
20. D
21. D
22. B
23. B
24. E
25. A
26. B
27. A
28. B
29. E
30. D

Team

1. $-\frac{5}{2}$

2. $\frac{840}{41}$

3. -2

4. 0.20

5. 357

6. 0

7. 729

8. –11

9. 0

10. 0.36

11. $\frac{8}{3}$ or $2.\overline{6}$

12. 1

13. -1

14. 70.5 (or 70.5 units)

15. $\frac{-30}{1-3\cos\theta}$

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Pre-Calculus Individual Solutions

1) $\dfrac{3x^2+3x-4}{x^3-x^2-x-2}=\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+x+1}$,

$3x^2+3x-4=Ax^2+Ax+A+Bx^2+Cx-2Bx-2C$,

$3x^2=Ax^2+Bx^2$

$3x=Ax-2Bx+Cx$, A=2, B=1, C=3, 1+2+3 = 6 **D**

$-4=A-2C$

2) $\lim\limits_{x\to 2}\dfrac{5x^2-3x-14}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(5x+7)}{x-2}=\lim\limits_{x\to 2}(5x+7)=17$, **D**

3) Law of cosines: $a=\sqrt{b^2+c^2-2bc\cos(A)}=13.9$ **E**

4)
$$\begin{array}{r} 2x+18 \\ x^2-5x+6\overline{)\,2x^3+8x^2-22x-60} \\ \underline{2x^3-10x^2+12x} \\ 18x^2-34x-60 \\ \underline{18x^2-90x+108} \\ 56x-168 \end{array}$$

$y=2x+18$

$y=18$

C

5) $500020=102020\left(1+\dfrac{.045}{4}\right)^{4t}$, $t=36$, **D**

6) Odd function: $f(-x)=-f(x)$, and symmetric to origin.
I, II, V, VI - True **C**

7) $\dfrac{.40oz\text{ Boric Acid}}{1oz\text{ Solution}}\times 30oz\text{ Solution}=12oz\text{ Boric Acid present}$

$\dfrac{12\text{ oz Boric Acid}}{(30+\text{x})\text{ oz Solution}}=.35=\dfrac{7}{20}$, $12=.35(x+30)$, $\dfrac{240}{7}=x+30$, $x=\dfrac{240}{7}-30=\dfrac{30}{7}$

A

8) $LR=\dfrac{2b^2}{a}$, $e=\dfrac{c}{a}$, $e=\dfrac{2}{1}$, $c=6$, $e=\dfrac{6}{a}$, $2=\dfrac{6}{a}$, $a=3$

$c^2=a^2-b^2$, $6^2=9-b^2$, $b^2=27$, **A**

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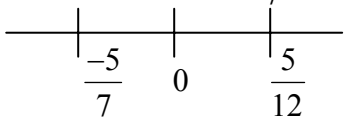
9) $\sqrt{x^2-36}$, $x^2-36\geq 0$, $x^2\geq 36$, $x\geq 6, x\leq -6$,
$\log(x^2+x-20)$, $x^2+x-20>0$, $(x+5)(x-4)>0$, $x<-5, x>4$,
Together the domain is $(-\infty,-5)\cup(4,\infty)$ **, E**

10) Period of cos(2x) is π. Period of cos(3x) is $\frac{2\pi}{3}$. Taking two periods of cos(2x) gives 2π and 3 periods of cos(3x) also gives 2π (essentially this is the least common multiple). Therefore, the period is 2π, **B**

11) $\left|e^{\frac{5\pi i}{2}}-e^{\frac{3\pi i}{2}}\right|=|i+i|=|2i|=2$, **E**

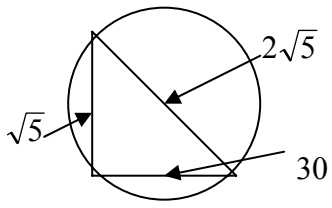
12) $12x-5=0$ $\qquad$ $7x+5=0$
$x=\frac{5}{12}$ $\qquad$ $x=\frac{-5}{7}$



$12x-5=3+7x+5$ $\qquad$ $-12x+5=3+7x+5$ $\qquad$ $-12x+5=3-7x-5$
$5x=13$ $\qquad$ $-19x=3$ $\qquad$ $-5x=-7$ $\qquad$ **B**
$x_1=\frac{13}{5}$ $\qquad$ $x_2=\frac{-3}{19}$ $\qquad$ ~~$x_3=\frac{7}{5}$~~

13) $A=\frac{1}{2}(62)(74)\sin 132$, $\qquad$ $c^2=(62^2)+(74^2)-2(62)(74)\cos 132$
$A_1=1704.77$ $\qquad$ $c=124.338$
$74^2=62^2+124.34^2-2(62)(124.34)\cos\theta$
$\theta=26.25$
$127-26.25=100.75$ $\qquad$ $A=\frac{1}{2}(54)(125.13)\cos 100.75$
$A_1+A_2=5003$ $\qquad$ $A_2=3298.21$ **D**

14) $-4-4=\sqrt{6\times 20}\cos\theta$, $\theta=136.91$, $180-\theta=43.089$, $\cos\theta=\frac{2\sqrt{30}}{15}$, **A**

15) 

$2x=2\sqrt{5}$, $r=\sqrt{5}$
$\pi r^2=(\sqrt{5})^2\pi$, $A=5\pi$, **A**

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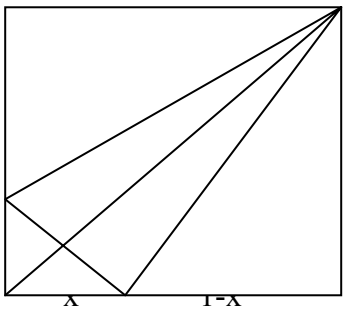
16) A right triangle is formed by the edge, diagonal of the cube, and a diagonal of a face. The hypotenuse is the diagonal of the cube which has length $\sqrt{3}x$. The adjacent side is the edge of the cube, which has length x. Therefore the angle is $\cos^{-1}\left(\frac{x}{\sqrt{3}x}\right)=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)=55°$, **C**

17) $\left(\frac{e^x+e^{-x}}{2}\right)^2+\left(\frac{e^x-e^{-x}}{2}\right)^2=\frac{2e^{2x}-2e^{-2x}}{4}=\frac{e^{2\pi i}-2e^{-2\pi i}}{2}=1$, **B**

18) $e^x-\frac{20}{e^x}=8,\ e^{2x}-20=8e^x,\ e^x=x,\ (x-10)(x+2)=0,\ e^x=10,\ x=\ln 10$, **B**

19) $\sqrt{(9^2)+(5^2)}=\sqrt{106}$, **E**

20) 

1

$1^2+(1-x)^2=\frac{2x}{\sqrt{2}},\ 1+1-2x+x^2=\frac{4x^2}{2},$

$x^2+2x-2=0,\ x=-1\pm\sqrt{3}$

$A=\left(\frac{-2+2\sqrt{3}}{\sqrt{2}}\right)^2\times\frac{\sqrt{3}}{4},\ A=2\sqrt{3}-3$, **D**

21) $cis\frac{\pi}{4}=\left(e^{\frac{i\pi}{4}}\right)^{4i}=e^{-\pi}$, **D**

22) $e^{0.2i}-3e^{0.6i}=\cos(0.2)-3\cos(0.6)+i(\sin(0.2)-3\sin(0.6))=-1.496-1.495i$,
$\sqrt{1.496^2+1.495^2}=2.1$, **B**

23) Quadrant II, **B**

24) All are members, **E**

25) $\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x=(2\cos^2 x-1)(\cos x)-2\sin^2 x\cos x$
$2\cos^3 x-\cos x-2\cos x(1-\cos^2 x)=2\cos^3 x-\cos x-2\cos x+2\cos^3 x$
$4\cos^3 x-3\cos x$, M+N = 1 **A**

26) $3=d,\ 3\pi=d\pi$, **B**

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27) $t = \dfrac{\ln 2}{.03} = \dfrac{\log 2}{.03 \log e}$ $-y = -x - 5$ $y = x + 5$ $-y\sqrt{3} = -x + 3\sqrt{3}$ $y = \dfrac{\sqrt{3}}{3}x - 3$

$\tan\theta = \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$, $\tan\theta = \dfrac{m_2 - m_1}{1 + m_2 m_1}$, $\theta = 15$, **A**

28) $\sin x = \dfrac{1}{2}\sec x$, $\left(\dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2}\right)$, $\sin x \cos x = \dfrac{1}{2}$, $\left(\dfrac{5\pi}{4}, -\dfrac{\sqrt{2}}{2}\right)$

$\sin 2x = 1$, $2x = 90 + 360n$, $x = 45 + 180n$, $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$, $y = \sin\dfrac{\pi}{4}, \sin\dfrac{5\pi}{4}$,

$m = \dfrac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{\frac{\pi}{4} - \frac{5\pi}{4}} = \dfrac{-\sqrt{2}}{\pi}$, $y - \dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{2}}{\pi}\left(x - \dfrac{\pi}{4}\right)$, $y = \dfrac{-\sqrt{2}}{\pi}x + \dfrac{3\sqrt{2}}{4}$, **B**

29) $A = Pe^{rt}$, $100 = 50e^{.03t}$, $2 = e^{.03t}$, $\ln 2 = .03t$, $t = \dfrac{\ln 2}{.03} = \dfrac{\log 2}{.03 \log e}$, **E**

30) $\cos\left(\dfrac{5\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$ **D**