2003 Palm Harbor Invitational Pre-Calculus Individual Test Answers

1. A
2. B
3. A
4. C
5. A
6. C
7. D
8. D
9. D
10. A
11. B
12. C
13. A
14. C
15. D
16. D
17. E
18. C
19. B
20. B
21. B
22. D
23. D
24. C
25. B
26. A
27. C
28. C
29. E
30. B

Solutions for the 2003 Palm Harbor Invitational Pre-Calculus Individual Round

1. A

$f(g(x)) = 4(x-3)^2 + 3(x-3) + 6$

$= 4x^2 - 24x + 36 + 3x - 9 + 6$

$= 4x^2 - 21x + 33$

2. B

$(x+42)(x-4) = x^2 + 38x - 168$

Because the x^2 coefficient of $0 = 3x^2 + kx - 504$ is 3, we multiply $x^2 + 38x - 168$ by 3:

$3x^2 + 114x - 504$

Hence, $k = 114$.

3. A

	5	3	0	4	1	−7
-1		-5	2	−2	−2	1
	5	-2	2	2	−1	−6

The remainder is –6.

4. C

$|(3i+4)(7i-9)|$

$|-21-27i+28i-36|$

$|-57+i|$

$\sqrt{(-57)^2+(1)^2}$

$5\sqrt{130}$

5. A

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 12 \\ 1 & 1 & 1 & 9 \\ 3 & 2 & -1 & 11 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 1 & 1 & 1 & 9 \\ 3 & 2 & -1 & 11 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 1 & 1 & 1 & 9 \\ 0 & -1 & -4 & -16 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 1 & 1 & 1 & 9 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

6. C

$\lfloor 345 \div 5 \rfloor = 69$

$\lfloor 345 \div 25 \rfloor = 13$

$\lfloor 345 \div 125 \rfloor = 2$

$69 + 13 + 2 = 84$

7. D

DeMoivre's Theorem states: $z^n = r^n \text{cis}(n\theta)$

Hence, $\sqrt[6]{z} = z^{\frac{1}{6}} = r^{\frac{1}{6}} \text{cis}\left(\frac{\theta}{6}\right)$

8. D

32 cis 250° has equivalent forms:

32 cis 610°

32 cis 970°

32 cis 1330°

32 cis 1690°

Using DeMoivre's Theorem a fifth root may be found from these:

2 cis 50°

2 cis 122°

2 cis 194°

2 cis 266°

2 cis 338°

9. D

$i^{2003} = i^3$

Hence, the answer is $-i$.

10. A

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 2 & 3 \\ 6 & 7 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 3 \\ 7 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 4 & 3 \\ 6 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 4 & 2 \\ 6 & 7 \end{vmatrix}$$

$$= -19\vec{i} + 14\vec{j} + 16\vec{k}$$

11. B

$3^2 + 1.25^2 = 3.25^2$

Hence,

$$\text{Area} = \frac{1}{2} \times 3 \times 1.25$$

$$\text{Area} = 1.875$$

12. C

$$\begin{array}{r} x+6 \\ x-3\overline{)x^2+3x+4} \\ \underline{x^2-3x} \\ 6x+4 \\ \underline{6x-18} \\ 22 \end{array}$$

Hence, it is $y = x + 6$.

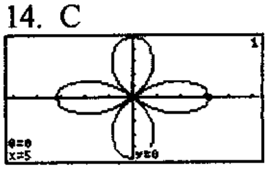
13. A

$x^2 - 6x + y^2 + 12y = -20$

$(x-3)^2 + (y+6)^2 = 25$

radius is 5, so area is 25π

14. C



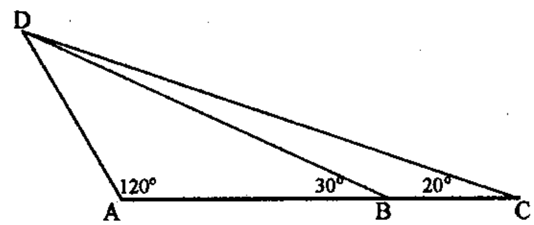
15. D

$x^2 - 6x + y^2 + 12y = -20$

$(x-3)^2 + (y+6)^2 = 25$

radius is 5, so area is 25π

16. D



$$\frac{DC}{\sin 150°} = \frac{78}{\sin(180° - (150° + 20°))}$$

$$DC \approx 224.592048843$$

$$\frac{DC}{\sin 120°} = \frac{AC}{\sin(180° - (120° + 20°))}$$

$$AC \approx 166.698327323$$

$$\frac{DC}{\sin 120°} = DA$$

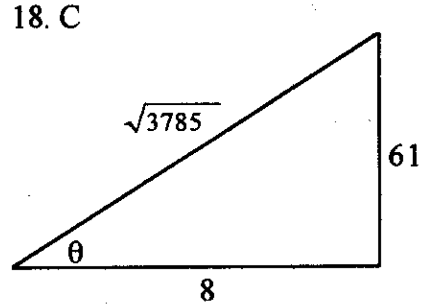
$$DA \approx 88.6983273231$$

$$AC + DA + DC \approx 479.99$$

17. E

All the functions are the same. $\sin\theta$ is an odd function.

18. C



$$\sin\theta = \frac{61}{\sqrt{3785}}$$

19. B

$B^2 - 4AC = 4^2 - 4(6)(8)$

This is negative and, hence, an ellipse.

20. B

Vertex:

$$\frac{-b}{2a} = \frac{-3}{0.5}$$
$$= -6$$

$0.25(-6)^2 + 3(-6) + 2 = -7$

The vertex is at (-6, -7). Because the $a = 0.25$ and $a = \frac{1}{4p}$, the focal length is 1. The directrix is the line $y = -8$. Points A and B have y coordinate -6:

$$-6 = 0.25x^2 + 3x + 2$$
$$-24 = x^2 + 12x + 8$$
$$0 = x^2 + 12x + 36$$
$$0 = (x+4)(x+8)$$
$$x = -4, -8$$

Points A and B have coordinates (-4, -6) and (-8, -6).

$$\frac{dy}{dx} = 0.5x + 3$$

$0.5(-4) + 3 = 1$

$0.5(-8) + 3 = -1$

The lines tangent to the curve at A and B are:

$y = x - 2$

$y = -x - 14$

Because y = -8 at the directrix:

$-8 = x - 2$

$x = -6$

$-8 = -x - 14$

$x = -6$

Both lines intersect the directrix at (-6, -8)

21. B

$\vec{a} \bullet \vec{b} = |a||b|\cos\theta$

Let

$a = \overline{CE}$

$b = \overline{CD}$

$\theta = \angle ECD$

$\vec{a} = (h-t)\vec{i} + (v-g)\vec{j}$

$\vec{b} = (h-t)\vec{i} + (s-g)\vec{j}$

$$(h-t)^2 + (s-g)(v-g)$$
$$= \sqrt{\left[(h-t)^2 + (s-g)^2\right]\left[(h-t)^2 + (v-g)^2\right]}\cos\theta$$

$$\cos\theta$$
$$= \frac{(h-t)^2 + (s-g)(v-g)}{\sqrt{\left[(h-t)^2 + (s-g)^2\right]\left[(h-t)^2 + (v-g)^2\right]}}$$

22. D

Scalar projection of $\vec{b}$ on $\vec{a}$ is $|\vec{b}|\cos\theta$, where θ is the angle between $\vec{b}$ and $\vec{a}$.

$5\cos 10^\circ$

23. D

The limit approaches 0 from the left, but does not approach anything from the right.

24. C

$$C^2 = A^2 + B^2 - 2AB\cos c$$
$$36^2 = 15^2 + 25^2 - 2(15)(25)\cos c$$
$$c \approx 126.49$$

$180 - 126.49 = 53.51$

25. B

$\sqrt{f(n)} = f(n-1)$

$f(n-1) = \sqrt{f(n-2)}$

$f(n-2) = \sqrt{\sqrt{f(n)}}$

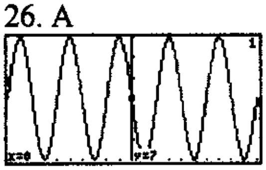
$f(n-2) = [f(n)]^{\frac{1}{2^2}}$

As $f(x)$ goes down, the square root is constantly being taken. Hence,

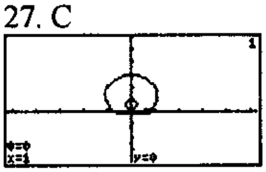
$f(n-a) = [f(n)]^{\frac{1}{2^a}}$

Because $\lim_{a\to\infty}[b]^{\frac{1}{2^a}} = 1$, as x approaches negative infinity, $f(x)$ approaches 1.

26. A



27. C



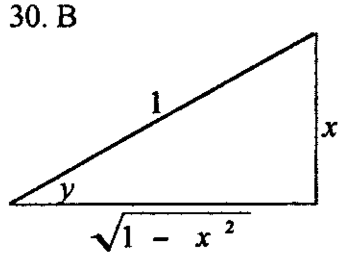
28. C

$$\lim_{x\to 7}\frac{x^2-4x-21}{x-7} = \lim_{x\to 7}\frac{(x-7)(x+3)}{x-7}$$
$$= \lim_{x\to 7}(x+3)$$
$$= 7+3$$
$$= 10$$

29. E

Because the determinant is 0, it has no inverse.

30. B



$$\frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$