PRE-CALC INVITATIONAL 2000 PHUHS

1 D
2 B
3 C
4 A
5B
6A
7B
8A
9C
10B
11D
12C
13B
14D
15A
16D
17 A
18D
19C
20C
21C
22C
23A
24E (ALL POSITIVE NUMBERS)
25B
26D
27E (3)
28A
29D
30A

Palm Harbor University Invitational
Pre Calculus Individual Solutions

1. D $m=(15-5)/(2-(-3))=2$ $\quad m_{\perp}=-1/2$ $\quad$ midpoint$(-1/2,10)$
$-1/2(x+1/2)=y-10 \Rightarrow y=-x/2+39/4$

2. B Total time: 4 hours 48 minutes 4+48/60 revolutions

$$4.8rev\times\frac{(4.5in)2\pi}{1rev}=\frac{216\pi}{5}in$$

3. C i) f(x) B^2-4AC $22^2-4(1)(-3360)=13{,}924 \Rightarrow + \Rightarrow$ 2 real roots

-7	6	29	-96	-35
	-42	91	35

ii) g(x) $(x+7)(6x^2-13x-5) \Rightarrow B^2-4AC\ (-13)^2-4(6)(-5)=289 \Rightarrow + \Rightarrow$ 3 real roots

| 6 | -13 | -5 | 0
---|---|---|---|---
5 | 1 | -13 | 65 | -151 | 130

$\underline{2}$	1	-8	25	-26
	2	-12	26

iii) h(x) $\Rightarrow (x-5)(x-2)(x^2-6x+13)$

| 1 | -8 | 25 | -26 | 0
---|---|---|---|---|---
5 | | -40 | 125 | -130

&

| 1 | -6 | 13 | 0
---|---|---|---|---
2 | | -12 | 26

B^2-4AC $(-6)2-4(1)(13)=-16 \Rightarrow$ 2 imag. Roots & 2 real roots

iv) B^2-4AC $38^2-4(9)(54)=-500 \Rightarrow$ 2 imag. Roots

4. A $v=r\Omega$ $8m/s=3m\Omega$ $\therefore \Omega=8/3 \Rightarrow 7m\ (8/3)=56/3m/s$

5. B Unit vector$=\frac{\vec{v}}{|\vec{v}|}$ $\quad |\vec{v}|=\sqrt{4^2+6^2+10^2}=\sqrt{152}=2\sqrt{38}$

$$\therefore \frac{4i}{2\sqrt{38}}+\frac{6j}{2\sqrt{38}}+\frac{10k}{2\sqrt{38}} \Rightarrow \frac{\sqrt{38}}{19}i+\frac{3\sqrt{38}}{38}j+\frac{5\sqrt{38}}{38}k$$

6. A $\frac{25!}{22!3!}x^{22}y^3 \Rightarrow 2{,}300x^{22}y^3$ $\quad \frac{25!}{15!10!}x^{10}y^{15} \Rightarrow 3{,}268{,}760x^{10}y^{15}$

$\therefore 2{,}300+3{,}268{,}760=3{,}271{,}060$

7. B $f(x+1)=(2x^2+32x+128)(x-4)$. $f(x)=(2(x-1)^2+32(x-1)+128)((x-1)-4)$
$f(x)=2x^3+18x^2-42x-490$ $\therefore$ sum of roots $=-b/a \Rightarrow -18/2=-9$

8. A

$$x+1/2\overline{)-6x^2+7x+5}\quad -6x+10$$
$$-6x^2-3x$$
$$10x+5$$
$$-(10x+5)$$
$$0$$

$$\frac{(-6x+10)(x+1/2)}{(x+1/2)} \Rightarrow (-6x+10) \qquad \therefore -6(-1/2)+10=13$$

9. C $B=\begin{bmatrix}1&2&0\\4&0&1\\9&2&3\end{bmatrix}$ $B^2=\begin{bmatrix}9&2&2\\13&10&3\\44&24&11\end{bmatrix}$ $\det(B^2)=64$

10. B $\sum_{n=1}^{x}n^3=\frac{x^2(x+1)^2}{4}$ $\therefore \frac{150^2\cdot151^2}{4}-\frac{4^2\cdot5^2}{4}=128{,}255{,}525$

11. D $f(x)=(2x^2+5x+6)^2$ $\therefore f'(x)=2(2x^2+5x+6)(4x+5) \Rightarrow f'(7)=9174$

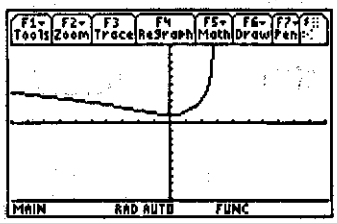
12. C $y=2x^2+5x+6 \quad \therefore 2(7)^2+5(7)+6=139 \Rightarrow m=\frac{906-139}{20-7}=59$

13. B 13^{453} Multiply the last 2 digits of each result by 13, and obtain the next two last digits. Repeat until a pattern develops.

$13^0=01$, $13^1=13$, $13^2=\ldots 69 \Rightarrow 69(13)=897 \Rightarrow 97(13)=1261$ etc..., $13^{19}=\ldots 77$, $13^{20}=\ldots 01 \Rightarrow$(pattern repeats) $453/20=22.65 \Rightarrow .65(20)=13 \quad 13^{13}=\ldots 53$

14. D $\cos 30^\circ=\frac{(3i-4j+12k)(-6i+2j+\lambda k)}{\sqrt{((-6)^2+2^2+\lambda^2)(3^2+(-4)^2+12^2)}} \Rightarrow \frac{\sqrt{3}}{2}=\frac{12\lambda-26}{13\sqrt{\lambda^2+40}} \Rightarrow \left(13\sqrt{3}\sqrt{\lambda^2+40}\right)^2=(24\lambda-52)^2$

$69\lambda^2-2496\lambda-17576=0 \Rightarrow \lambda=\frac{26\sqrt{4098}+1248}{69} \Rightarrow 26+4098+1248+69=5441$



(Note: $\lambda=\frac{-26\sqrt{4098}+1248}{69}$ is not a correct solution)

15. A $f(x)=\sqrt{x+1} \quad g(x)=\frac{2x^2}{3-x} \quad (f \circ g)=\sqrt{\frac{2x^2}{3-x}+1}$

Domain: $x<3$ or $(-\infty,3)$

16. D $(6)(\Omega)+(5)(7)+(-2)(-11)=111 \Rightarrow \Omega=9$

17. A 3000ft.=1000yd. $\tan 37^\circ = x/1000 \Rightarrow x=753.55 \approx 754$yd.

18. D The intersection of the planes can be $\cos^{-1}\left(\frac{(2)(4)+(-3)(-1)+(5)(1)}{\sqrt{(2^2+(-3)^2+5^2)(4^2+(-1)^2+1^2)}}\right) \approx 52^\circ$ represented by the vectors contained in them.

19. C $\tan\theta=4/5 \Rightarrow \theta \approx 38.7^\circ$

20. C When a plane intersects the cone in 1 point, a degenerate ellipse is formed.

21. C $(\log x)^2-6(\log x)=7 \therefore$ let $y=\log x \Rightarrow y^2-6y=7 \Rightarrow y=7$ or $-1 \Rightarrow 7=\log x$ and $-1=\log x$ $x=10{,}000{,}000$ and $1/10 \therefore (10{,}000{,}000)(1/10)=1{,}000{,}000$

22. C $\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}-\tan x \Rightarrow \frac{\sin^2 x}{\sin x(1-\cos x)}+\frac{1-2\cos x+\cos^2 x}{\sin x(1-\cos x)}-\tan x \Rightarrow \frac{2-2\cos x}{\sin x(1-\cos x)}-\tan x \Rightarrow$

$\frac{2(1-\cos x)}{\sin x(1-\cos x)} \Rightarrow \frac{2}{\sin x}-\frac{\sin x}{\cos x} \Rightarrow \frac{2\cos x-\sin^2 x}{\sin x \cos x}$

23. A $4\begin{vmatrix}3 & 8\\ 6 & 1\end{vmatrix}-9\begin{vmatrix}a & 8\\ 7 & 1\end{vmatrix}+2\begin{vmatrix}a & 3\\ 7 & 6\end{vmatrix}=297 \Rightarrow 4(3-6(8))-9(a-8(7))+2(6a-3(7))=297 \Rightarrow a=5 \therefore 2a=10$

24. E, All positive numbers $\log_2 \alpha = \log_4 \alpha^2 \Rightarrow y=\log_2\alpha \Rightarrow y=\log_4\alpha^2 \Rightarrow 4^y=\alpha^2 \Rightarrow 2^{2y}=\alpha^2 \Rightarrow (2^2)^y=(2^y)^2$ Equal for all real numbers, but can only take the log of a positive number.

25. B $r=17\csc\theta$ is a line

26. D $a=1/(4p) \Rightarrow p=1 \therefore a=1/4 \Rightarrow a(y-k)^2=(x-h) \Rightarrow (1/4)(y-7)^2=(x-4) \Rightarrow y^2-4x-14y+65=0$

27. E, 3 $f(x)=\frac{6x^2-3x-45}{2x+5} \Rightarrow f'(x)=\frac{(2x+5)(12x-3)-(6x^2-3x-45)(2)}{(2x+5)^2}=\frac{12x^2+60x+75}{4x^2+20x+25}=3$

28. A FJ/JK hyp./adj. $\Rightarrow \sec\theta$

29. D $y=ax^2+bx+c \Rightarrow 8=a+b+c$, $47=16a+4b+c$, $103=36a+6b+c$, solving simultaneous equations yields $a=3$, $b=-2$, $c=7$. $a+b=1$

30. A $4(9)+7G=50 \Rightarrow G=2 \qquad 4F+7(3)=28 \Rightarrow F=7/4 \quad G-F=1/4=.25$