February 2005 Palm Harbor Invitational Geometry Individual Test

Let choice E) NOTA denote "None Of These Answers."
Figures are NOT drawn to scale.

1. Given: ΔABC with $AB = 10$ and $BC = 12$ Let $\overline{AD}$ bisect $\angle A$, with point D on segment $\overline{BC}$. If $CD = 4$, then what is the length of side $\overline{AC}$?

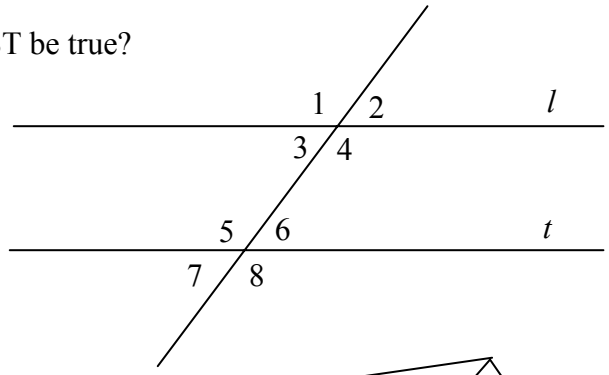
   A. 4 B. 5 C. 6 D. Cannot be determined E. NOTA

2. Two sides of a triangle measure 12 inches and 5 inches. Which of the following lengths could NOT be the measure of the third side?

   A. 7.1 inches B. $\sqrt{127}$ inches C. 13 inches D. $\sqrt{289}$ inches E. NOTA

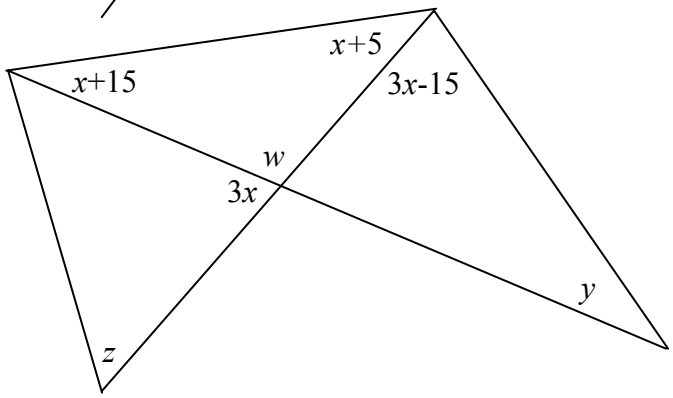
3. In the given figure, which of the following MUST be true?

   a. line l || line t
   b. $m\angle 2 = m\angle 6$
   c. $\angle 5$ & $\angle 6$ are supplementary
   d. All of the above
   e. NOTA



4. In the figure to the right, what is the value of y?

   A. 20 B. 45
   C. 65 D. 75
   E. NOTA



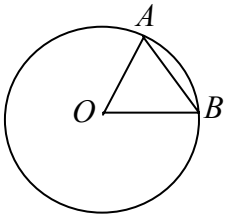
5. Given: All gazzles are green.
   All things green live on Gazzle I.

   Which of the following is NOT true?
   I. If it lives on Gazzle I, then it is green.
   II. If it is a gazzle, then it lives on Gazzle I.
   III. If it is not green then it doesn't live on Gazzle I.

   A. I. only B. I. & II. only C. I. & III. only D. I., II., & III. E. NOTA

6. Given: Circle O, $\overline{OA} = 7$, $m\angle AOB = 90°$. Find $\overline{AB} \times \overline{OA} \times \overline{OB}$.

   A. $49\sqrt{2}$ B. 686 C. $303\sqrt{2}$ D. $343\sqrt{2}$ E. NOTA



7. To the nearest tenth, what is the volume of a cone that has a slant height that makes an angle of $60°$ with the plane of the base and has a height of 10?

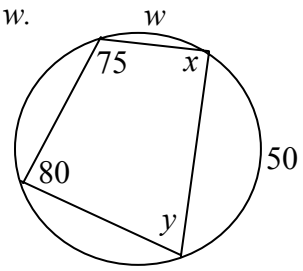
   A. 261.8 B. 349.1 C 785.4 D. 1047.2 E. NOTA

8. A solid metal cylinder was dropped in a pit of lava. It initially had a radius of 6 cm and a height of 18 cm. When it was rescued, its radius was 9 cm but its volume had remained unchanged. What was the new height of the cylinder?
   A. 6 cm B. 8 cm C. 10 cm D. 15 cm E. NOTA

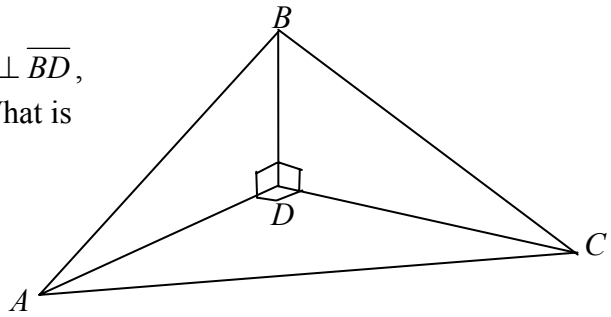
9. Given: ΔABC with side lengths of 20, 30, and 45. What is the ratio of the length of the longest altitude to the length of the shortest altitude of ΔABC?
   A. $\frac{9}{4}$ B. $\frac{3}{2}$ C. $\frac{9}{8}$ D. Cannot be determined E. NOTA

10. Point X is the point of intersection of the angle bisectors of ΔABC. X is known as the …
    A. circumcenter B. orthocenter C. centroid D. bisectioner E. NOTA

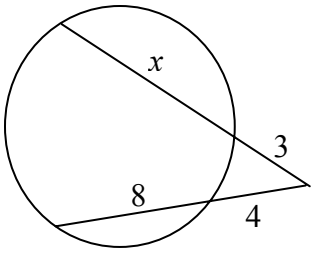
11. Given that the quadrilateral is inscribed in the circle. Find $x + y + w$.
    A. 205 B. 235 C. 285 D. 315 E. NOTA



12. Given: triangular pyramid ABCD, $\overline{AD} \perp \overline{BD}$, $\overline{CD} \perp \overline{BD}$, & $\overline{AD} \perp \overline{CD}$. If $\overline{AB} = 25$, $\overline{BC} = 17$, & $\overline{BD} = 15$ What is the length of $\overline{AC}$?
    A. $2\sqrt{7}$ B. $\sqrt{506}$
    C. $4\sqrt{29}$ D. $3\sqrt{14}$
    E. NOTA

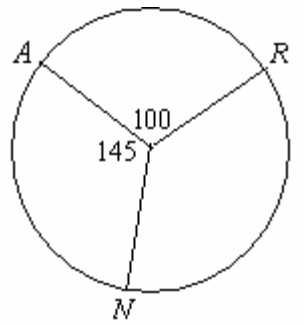


13. In the figure to the right, what is the value of x?
    A. 7 B. 10 C. 12 D. 13 E. NOTA

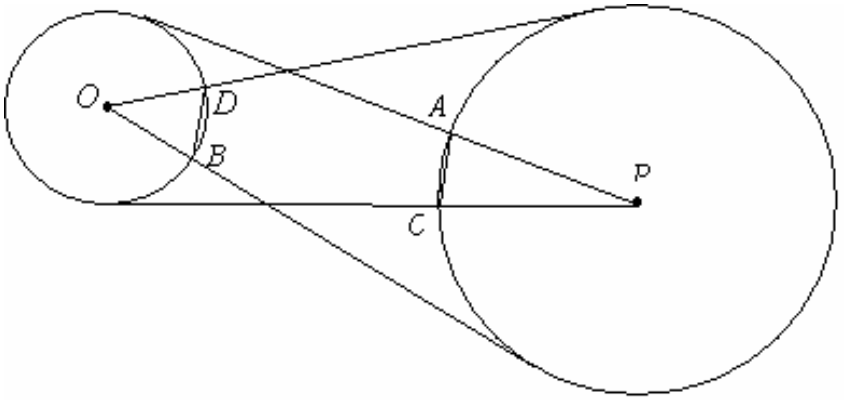


14. Marcus is building a birdhouse in a tree that is 12 feet tall. His ladder is 15 feet long. How far from the base of the tree must Marcus place the ladder so that it reaches the top of the tree?
    A. 4.5 ft B. 7.5 ft C. 9.0 ft D. 15 ft E. NOTA

15. The measures shown are the degree measures of two central angles. What is the measure of arc ANR?
    A. 115 B. 260 C. 290 D. Cannot be determined
    E. NOTA



16. In the figure to the right, circle *O* has a radius that is one half the radius of circle *P*. Which of the following MUST be true about the lengths of $\overline{AC}$ & $\overline{BD}$ (assume the segments emanating from the center of each circle are tangents)?

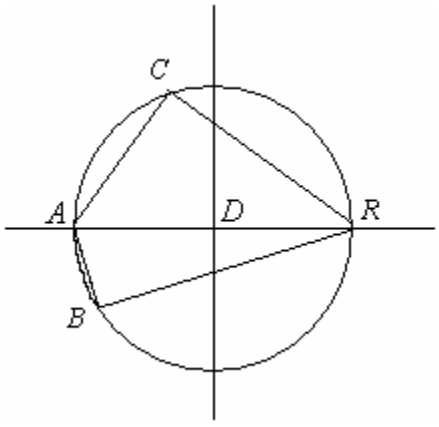


A. $\overline{AC} > \overline{BD}$
B. $\overline{AC} = \overline{BD}$
C. $\overline{AC} < \overline{BD}$
D. The comparative lengths cannot be determined from the information given.
E. NOTA

17. In ΔABC & ΔDCE it is known that $\overline{AB} \cong \overline{DC}$, & $\overline{CB} \cong \overline{EC}$. Which of the following would be sufficient to know that $\Delta ABC \cong \Delta DCE$?
A. $\angle A \cong \angle D$ B. $\angle BCA \cong \angle CED$ C. $\angle BCA$ & $\angle D$ are right angles
D. All of the above E. NOTA

18. Given an equilateral triangle inscribed in a circle and a square circumscribed about the same circle. What is the ratio of the area of the given square to the area of the given circle?
A. $\frac{9}{4}$ B. $\frac{3\sqrt{3}}{2}$ C. $\frac{9\sqrt{3}}{4}$ D. $\frac{16\sqrt{3}}{9}$ E. NOTA

19. In the figure to the right, D is the center of the circle, quadrilateral ABRC is inscribed in circle D, and $\angle ADR = 180°$. If $\overline{AC} = 3\overline{AB}$, what is $m\angle ACR$?
A. 60 B. 85 C. 90 D. 120 E. NOTA



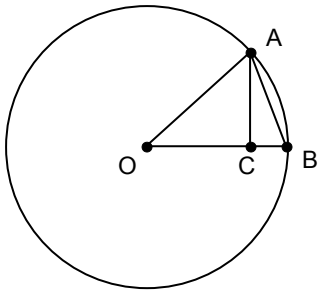
20. In a certain convex polyhedron, the number of edges is twice the number of vertices, while the number of faces is 4 less than the number of edges. What is the sum of the number of faces, edges, and vertices?
A) 22 B) 24 C) 26
D) 28 E) NOTA

21. Planet X has a diameter of 10,000 miles. When Planet X rotates around its sun, Inferno, the distance from the center of the planet to the center of Inferno is always 70 million miles. To 2 significant digits and in scientific notation, how much area does Planet X sweep out in one revolution around its sun Inferno? (Treat the planet and the sun as circles in the same plane.)
A) 4.4×10^8 B) 4.4×10^9 C) 4.4×10^{10} D) 4.4×10^{12} E) NOTA

22. Melissa is trying to construct a triangle with sides 6, 7, and 8 using a compass and a straightedge. The first thing she does is to draw a line segment $\overline{AB}$ of length 6. What would be the most logical next step in this construction?
A) Draw one circular arc of radius 7 centered at one endpoint of $\overline{AB}$, then an arc of radius 8 centered at an arbitrary point on that arc.
B) Draw two circular arcs, one of radius 7 and one of radius 8, both centered at point A.
C) Draw two circular arcs, one of radius 7 and one of radius 8, centered at A and B, respectively.
D) Draw two circular arcs of radius 7 centered at opposite endpoints of $\overline{AB}$; then do the same for two circular arcs of radius 8.
E) NOTA

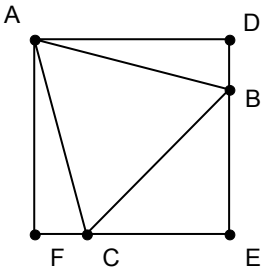
23. A trapezoid is inscribed in a circle with radius 6. One of the parallel sides of the trapezoid is a diameter of the circle, while the other has length 8. What is the height of the trapezoid?
A) $2\sqrt{3}$ B) 4 C) $2\sqrt{5}$ D) $\frac{9}{2}$ E) NOTA

24. In this diagram, the radius of the circle with center O is 25 units, AC measures 7 units, and AC is the altitude to side OB. What is the length of side AB?
A) $5\sqrt{2}$ B) $2\sqrt{13}$ C) $3\sqrt{6}$ D) $7\sqrt{2}$ E) NOTA



25. Which of the following statements is not true?
A) If two lines are not parallel and they do not intersect, then they are not in the same plane.
B) The sum of the exterior angles of a regular dodecagon is equal to the sum of the exterior angles of a regular nonagon.
C) If line A is perpendicular to line B and line B is perpendicular to line C, then line A is parallel to line C.
D) Any three noncollinear points uniquely determine a plane.
E) NOTA

26. Equilateral triangle ABC is inscribed in square ADEF whose sides are length 1 so that the segments BD and CF are equal. What is the length of one side of triangle ABC, to 2 decimal places?



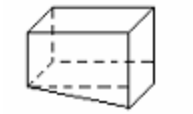
A) 1.02 B) 1.03 C) 1.04 D) 1.05 E) NOTA

27. Match each of the following shapes to the correct description of its area.

1) Circle with circumference 1	w) An integer
2) Square with diagonal 1	x) A terminating decimal
3) Isosceles right triangle with altitude to hypotenuse 1	y) A repeating decimal
4) A right triangle similar to a 3-4-5 right triangle with smallest side 1	z) An irrational number

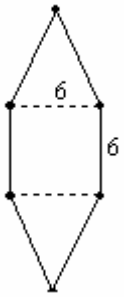
A) 1-z, 2-x, 3-w, 4-y B) 1-y, 2-x, 3-w, 4-z C) 1-z, 2-w, 3-x, 4-y

D) 1-y, 2-w, 3-x, 4-z E) NOTA



28. Taryn wants to have a swimming pool built for her, in the same shape as the figure to the right. If the perimeter around the top of the pool is 360 feet, the shallow side is 4 feet deep and the deep side is 8 feet deep, then what is the largest swimming pool Taryn can have, in terms of volume in cubic feet?

A) 48000 B) 48600 C) 51840 D) 57600 E) NOTA



29. Ryan decides to build a vegetable garden, but he doesn't know how much fence to buy to keep out the fuzzy bunnies. If the garden is laid out in the arrangement shown in the diagram (assume the top and bottom portions are congruent isosceles triangles, and the middle portion is a square) and must have an area of 60 square meters, how many meters of fencing are needed?

A) 20 B) 26 C) 32 D) 44 E) NOTA

30. Taryn, Channing, Jenn and Cody need to meet at one of their four houses to work on the Mu Alpha Theta scrapbook. Jenn's house is at (-4, -1), Taryn's is at (4, 2), Cody's is at (1, -7), and Channing's is at (-3, -5). The four students want to minimize the total distance traveled by everyone. Whose house must they meet at if they need to finish the scrapbook tonight, the night before they go to the State Convention?

A) Channing's B) Taryn's C) Cody's D) Jenns's E) NOTA (they gave up and went to the movies instead!)