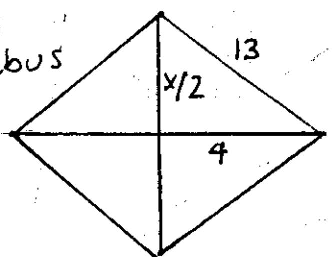
2004 Palm Harbor Invitational
Geometry Individual Solutions

① Total measure of all exterior angles $= 360°$ (for all polygons)
measure of 1 exterior angle $=$
$360°/5 = 72°$ $\boxed{C}$

② Distance from racquetball court $=$
$\sqrt{3^2+5^2} = \sqrt{34}$
Happiness $= \frac{8}{\sqrt{34}} \approx 1.4$ $\boxed{B}$

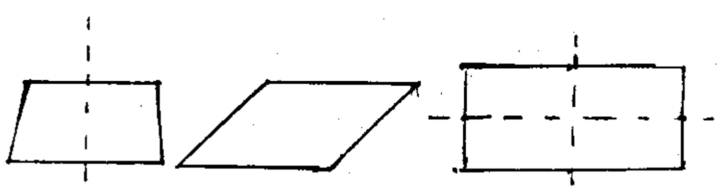
③

Diagonals of a rhombus are perpendicular



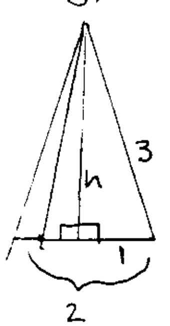
$4^2 + \left(\frac{x}{2}\right)^2 = 13^2$
$\left(\frac{x^2}{4}\right) = 153$
$x^2 = 612$ $x \approx 24.7$ $\boxed{D}$

④



1-y 2-x 3-z $\boxed{D}$

⑤ Let $x =$ the length of 1 side of one of the triangles
$8x = 3(4 \cdot 2)$ $8x = 24$ $x = 3$



$h^2 + 1^2 = 3^2$
$h = \sqrt{8} = 2\sqrt{2}$ $\boxed{D}$

⑥ multiply by the conjugate of the denominator
$\frac{5+\sqrt{5}}{7+\sqrt{3}} \cdot \frac{7-\sqrt{3}}{7-\sqrt{3}}$
$= \frac{35+7\sqrt{5}-5\sqrt{3}-\sqrt{15}}{49-3}$ $\boxed{C}$

⑦ Tangent lines are perpendicular to the line from the center to the point of tangency, so the slope $=$
$\frac{-1}{-2/3} = 3/2$ $\boxed{D}$

⑧ circumference of the 2 circular arcs $= 400 - 2(100) = 200$ m
Let r be the radius
$2\pi r \cdot \frac{1}{2} \cdot 2 = 200$
$r = \frac{100}{\pi}$ $\boxed{A}$

⑨ Area of segment = area of sector − area of triangle
Sector $= \frac{141°}{360°} \cdot \pi(3^2) \approx 11.07$
Height of triangle $= 3 - 2 = 1$
Base $= 2(\sqrt{3^2 - 1^2}) = 4\sqrt{2}$
Area of triangle $= \frac{1}{2}(4\sqrt{2})(1) = 2\sqrt{2}$
Area of segment $= 11.07 - 2\sqrt{2} \approx 8.24$
Total area of circle $= \pi(3^2) \approx 28.27$
$\frac{8.24}{28.27} = .2914 \approx 29\%$ $\boxed{C}$

⑩ Side lengths are:
a- $\sqrt{(-5-1)^2 + (2-4)^2} = \sqrt{40}$
b- $\sqrt{(-5-(-3))^2 + (2-0)^2} = \sqrt{8}$
c- $\sqrt{(1-(-3))^2 + (4-0)^2} = \sqrt{32}$
None are the same - Scalene $\boxed{C}$

(11) Books stacked together form a rectangular prism

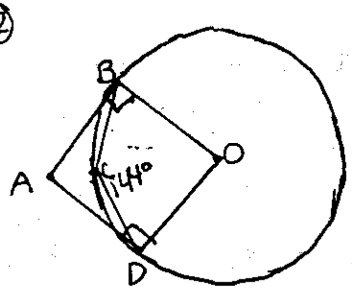
length = 6 width = 10

height = $2 \cdot 6 = 12$

Surface area $= 2(6 \cdot 10 + 6 \cdot 12 + 10 \cdot 12)$

$= 504$ $\boxed{A}$

(12)



- $\angle B$ and $\angle D$ are $90°$ because AB and AD are tangent to the circle
- measure of inscribed angle $= \frac{1}{2} \cdot$ central angle, so central angle corresponding to $\angle BCD = 2(144°) = 288°$
- $BOD = 360° - 288° = 72°$
- $BAD = 360° - 2(90) - 72° = 108°$ $\boxed{D}$

(13) #1: The angle opposite 92° must be 88°

The " " 95° must be 85°

#2: No information about the other 2 angles except they must add up to 180°

#3: The angle opposite the 88° must be 95°

The " " 88° must be 92°

∴ Quadrilaterals 1 and 3 must have angles of 92°, 95°, 85°, and 88° $\boxed{B}$

ow

(14) Radius of sphere: $4\pi r^2 = 1000$

$r = \sqrt{250/\pi}$

Area of circle $= \pi(\sqrt{250/\pi})^2 = 250$ $\boxed{C}$

(15) Centroid $= \left(\frac{7-11-1}{3}, \frac{4+8-12}{3}\right) = (-1, 0)$

At (9,0) you have to walk west

$\boxed{A}$

(16) A = If P is equilateral and regular, then P is equiangular TRUE

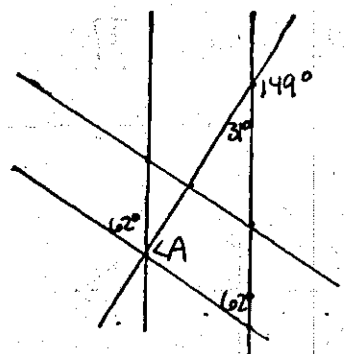
B = If P is regular, then P is equilateral TRUE

C = If P is equiangular and equilateral, then P is regular TRU

$\boxed{D}$ = If P is equiangular or regula then P is equilateral False

<counterexample; a rectangle is equiangular but not equilateral

(17)



$\angle A = 180° - 31° - 62° = 87°$ $\boxed{A}$

(18) Number of edges:

$\frac{20 \text{ triangles} \cdot 3 \text{ sides}}{2 \text{ (because edges overlap)}} = 30$

Total length of edges $= 30 \cdot 3 = 90$ [I

(19) Exterior angle $= 360/n$

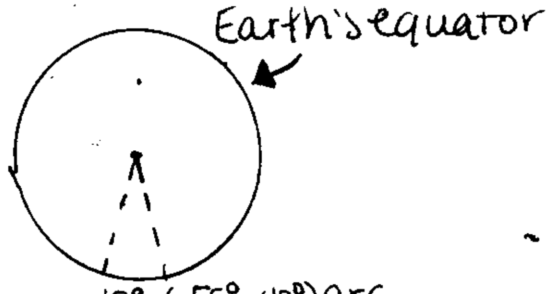
Interior angle $= 180(n-2)/n$

$180(n-2)/\cancel{n} = 3(360/\cancel{n})$

$180(n-2) = 360 \cdot 3$ $180n = 1440$ $n = 8$

(20) $\boxed{B}$ The negation of "All aliens are good at math" is "Some aliens a not good at math."

(21)



$15°$ ($55° - 40°$) arc

Circumference of earth $= \pi(8000)$
$= 8000\pi$

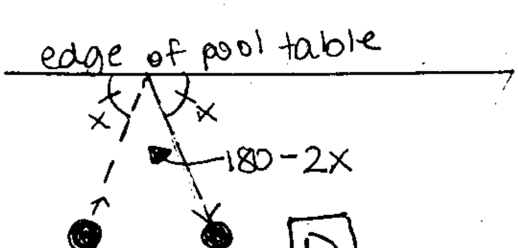
Distance along $15°$ arc $= \frac{15}{360}(8000\pi)$
$= \frac{1000\pi}{3}$ $\boxed{B}$

(22) $3x = 2y + 5$
$4y + 2x + 3 = 180 - 3x$
Solve the system of equations:
$3x - 2y = 5$
$5x + 4y = 177$ $\quad 6x - 4y = 10$
$11x = 187$ $\quad \underline{x = 17}$ $\quad \underline{y = 23}$ $\quad xy = 391$ $\boxed{A}$

(23) Area of one shaded part =
area of square - area of arc
* assume that square has side length 1
$1(1) - \pi(1^2)/4 \approx .215$
Area of both shaded parts $= 2(.215) = .429$
$.429/1(1) = .429 = 43\%$ $\boxed{C}$

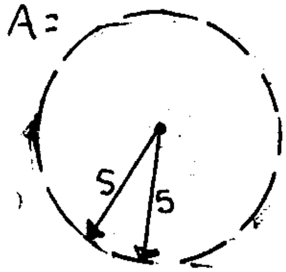
(24) Similar triangles: $ABC \sim ADE$ - ratio $2/3$
Let $AC = x$
$\frac{x}{x+2} = \frac{2}{3}$ $\quad 3x = 2x + 4$ $\quad x = 4$ $\boxed{B}$

(25)



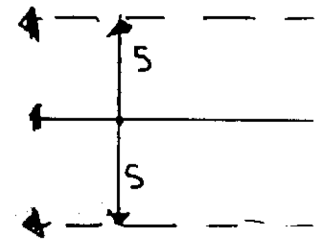
$\boxed{D}$

(26)
A =



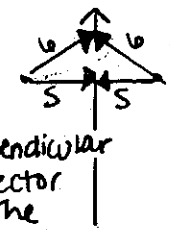
a circle with radius 5

B =



2 parallel lines

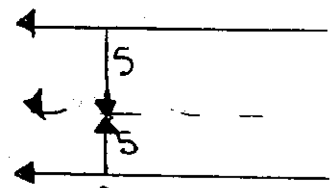
C =



perpendicular bisector of the segment between the lines

$\boxed{A}$

D =



line halfway between 2 lines

(27) for similar figures $P_1 + P_2$,
$\frac{\text{area } P_1}{\text{area } P_2} = \left(\frac{\text{side } P_1}{\text{side } P_2}\right)^2$

$\frac{\text{side A}}{\text{side B}} = \sqrt{.27} = .520$ $\quad \frac{\text{side B}}{\text{side C}} = \sqrt{.12} =$
Side A $= 1/.520/.346 = 5.56$ $\boxed{C}$

(28) x, y, are the lengths of sides
$xy = 30$ $\quad 2x + 2y = 34$ $\quad x + y = 17$
$x(17 - x) = 30$ $\quad x^2 - 17x + 30 = 0$ $(x-15)(x-2) =$
sides $= 2, 15$ $\boxed{A}$

(29)
Distance the hamster runs =
$1\text{min} \cdot \frac{1\text{hr}}{60\text{min}} \cdot \frac{2\text{mi}}{\text{hr}} \cdot \frac{5280\text{ft}}{\text{mi}} \cdot \frac{12\text{in}}{\text{ft}} = 2112$ ft
Distance of 1 revolution
$= 2\pi(6) = 12\pi$ in
Number of revolutions =
$2112/(12\pi) = 56.02$ $\approx \boxed{56}$ $\boxed{C}$

(30) Total surface area $= 2\pi r^2 + 2\pi rh$
If $h = r$, SA $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$
$\boxed{D}$