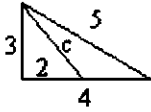


Geometry Answers

Individual	Team
1) D	1) 84
2) D	2) 798
3) D	3) 126°
4) C	4) $\frac{2}{3}$
5) B	5) 1600
6) A	6) 6421
7) A	7) 96
8) C	8) (2, 4)
9) C	9) 1800π
10) B	10) $\frac{4\sqrt{7}}{7}$
11) C	11) 16
12) E	12) π
13) A	13) 28
14) C	14) 364
15) B	15) $10\sqrt{3}$
16) A	
17) A	
18) C	
19) C	
20) B	
21) A	
22) D	
23) B	
24) C	
25) C	
26) D	
27) D	
28) B	
29) E	
30) C	

- 1) **Answer: D**
 $x^2 + (x+1)^2 = (x+9)^2$
 $x^2 + x^2 + 2x + 1 = x^2 + 18x + 81$
 $x^2 - 16x - 80 = 0 \quad x = -4, 20$
 x must be positive so $x = 20$

- 2) **Answer: D**
 3) **Answer: D**



- $3^2 + 2^2 = c^2 \quad c = \sqrt{13}$
- 4) **Answer: C**
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
 $d = \sqrt{(1-1)^2 + (2-(-1))^2 + (3-(-1))^2} = 5$

- 5) **Answer: B**
 i. true: 9° and 81°
 ii. false: $x^2 = 180 - x$
 $x^2 + x - 180 = 0$ is not factorable
 iii. false: $27w + 27x + 27y + 27z = 360$
 $27(w + x + y + z) = 360$
 $\frac{360}{27}$ is not an integer
 iv. true: a regular pentagon with where all five angles measure 108° .

- 6) **Answer: A**
 $15 \cos \theta = 14.4889$
 $\theta = 15^\circ$
 number of sides = $\frac{360^\circ}{2(15^\circ)} = 12$

- 7) **Answer: A**
 volume of cylinder = $\pi(3^2)(8) = 72\pi$
 $\frac{12\pi}{72\pi} = \frac{1}{6} \quad \frac{1}{6}(360^\circ) = 60^\circ$

- 8) **Answer: C**
 $\sqrt{3}(2y\sqrt{3} - 6) = y \quad 6y - 6\sqrt{3} = y$
 $y = \frac{6\sqrt{3}}{5} \quad C = \frac{6\sqrt{3}}{5} \cdot \frac{1}{2\sqrt{3}} = \frac{12}{5}$

- 9) **Answer: C**
 $x^2 + y^2 - 8x = 65$

$(x-4)^2 + y^2 = 81 \quad \text{center: } (4, 0)$

$x^2 + y^2 - 16x - 10y = -73$

$(x-8)^2 + (y-5)^2 = 16 \quad \text{center: } (8, 5)$

The distance between the centers is $\sqrt{41}$ or 6.403. The sum of the radii is 13, which is greater than the distance between, meaning that the circles must intersect in two places.

- 10) **Answer: B**
 main diagonal of the cube =
 $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3} = \text{diameter of the sphere}$
 $a\sqrt{3} = 6 \quad a = 2\sqrt{3}$
 surface area = $6a^2 = 6(2\sqrt{3})^2 = 72$

- 11) **Answer: C**
 DEF: SAS GHI: SSA JKL: AA
 only DEF and JKL are valid

- 12) **Answer: E**
 It depends on the central angle of AB

- 13) **Answer: A**
 diagonals = $\frac{n(n-3)}{2}$
 sum of interior angles = $180(n-2)$
 $\frac{n(n-3)}{2} = 180(n-2) - 1$
 $n = 361 \quad n = 400$ (nearest hundred)

- 14) **Answer: C**
 1. $\sqrt{5}^2 + \sqrt{11}^2 = 4^2$ right (z)
 2. $21^2 + 23^2 < 32^2$ obtuse (y)
 3. $16^2 + 17^2 > 23^2$ acute (x)

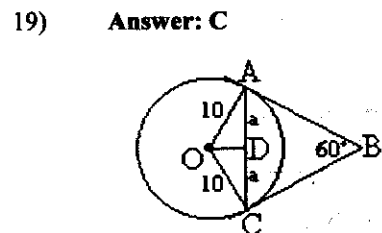
- 15) **Answer: B**
 The regular hexagon is made up of 6 equilateral triangles with areas of $\frac{1}{6}$. The shaded region is a third of the equilateral triangle, so its area is $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

- 16) **Answer: A**
 Slopes of sides = $2, 2, \frac{1}{8}, \frac{1}{8}$
 Opposite sides are parallel, but the slopes are not perpendicular; parallelogram but not rectangle.
 Side lengths = $\sqrt{45}, \sqrt{45}, \sqrt{65}, \sqrt{65}$
 Side lengths are not the same, thus not a rhombus.

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17) **Answer: A**
 $V = \frac{1}{3} Bh$ $V = \frac{1}{3}(16)(2\sqrt{2}) = 15.08\dots$

18) **Answer: C**
 AD = 8
 let BD = x
 $6x = 8^2$ $x = \frac{32}{3}$
 $AB = \sqrt{8^2 + \left(\frac{32}{3}\right)^2} = 13\frac{1}{3}$



$120^\circ, m\angle AOD = 60^\circ$
 $\frac{2a}{\sqrt{3}} = 10$ $2a = 10\sqrt{3} = 17.32\dots$

20) **Answer: B**
 $y^2 = 16 - x^2$ is the graph of a circle with radius 4 centered at (0, 0). The path traveled is one-fourth of the circumference.
 $\frac{1}{4}(2\pi(4)) = 2\pi$

- 21) **Answer: A**
 i. If $2 + 2 \neq 4$, then $3 + 3 \neq 5$.
 false \rightarrow true true
 ii. If $3 + 3 = 5$, then $2 + 2 = 4$.
 false \rightarrow true true
 i. If $3 + 3 \neq 5$, then $2 + 2 \neq 4$.
 true \rightarrow false false

22) **Answer: D**
 Let A be the area of the entire triangle.
 Area of trapezoidal region =
 $\frac{25}{36}A - \frac{16}{36}A = \frac{9}{36}A$
 Area of triangular region = $\frac{1}{36}A$
 ratio = $\frac{1}{9}$

- 23) **Answer: B**
 flip over y-axis: (-2, 4)
 rotate 90° clockwise: (4, 2)
 shift two units downward: (4, 0)

24) **Answer: C**
 Let r be the radius of the small circles and R be the radius of the large circle.

$R = r + \frac{2r}{\sqrt{3}} = 1r = 2\sqrt{3} - 3$

25) **Answer: C**
 centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$(3, 5) = \left(\frac{1 + 3 + x_3}{3}, \frac{3 + 4 + y_3}{3}\right)$

$(x_3, y_3) = (5, 8)$

26) **Answer: D**
 $5^2 + r^2 = 15^2$ $r = \sqrt{200}$
 area = $\pi r^2 = 200\pi$

27) **Answer: D**
 $\frac{x + y}{2} = 24.5$ $x + y = 49$ $y = 49 - x$

$\sqrt{xy} = 6\sqrt{10}$ $xy = 360$
 $x(49 - x) = 360$

$x^2 - 49x + 360 = 0$ $x = 9, 40$

$\sqrt{9^2 + 40^2} = 41$

$\frac{1}{41} = 0.0243\dots$

28) **Answer: B**
 $8(8 + 4) = 6(6 + 2r)$ $2r = 10$ $r = 5$

- 29) **Answer: E**
 A plane cannot intersect any of them in exactly three points.

30) **Answer: C**
 Area of square = $(r\sqrt{2})^2 = 2r^2$
 Area of hexagon = $6\left(\frac{r^2\sqrt{3}}{4}\right) = \frac{3r^2\sqrt{3}}{2}$

ratio = $\frac{3\sqrt{3}}{4}$