Palm Harbor Invitational – February 2005 Calculus Team Question # 1

Given: $f(x) = \sin(Ax)$, $A > 0$
The absolute area bounded by $f(x)$ and the *x*-axis is known to be 12 over the domain $[0, A \cdot \pi]$.
Find the exact value of *A*.

Palm Harbor Invitational – February 2005 Calculus Team Question # 2

For the given problem, *A*, *B*, and *C* are all interrelated and are all positive values.

Given: $A = \int_0^C \frac{dx}{x^2 + C^2}$

$B = \frac{d}{dx}\left[x^A\right]$ when $x = 2$

$C =$ The *x*-coordinate of the first point of inflection of $y = \sin(Bx)$ after $x = 0$.

Find the exact value of $\int_0^C B\cos(Ax)dx$

Palm Harbor Invitational – February 2005 Calculus Team Question # 3

Find the exact area enclosed within the graphs:
$y \le x + 2$, $y \le -x + 3$, $y \ge x^2$, and $y \ge 4x$

Palm Harbor Invitational – February 2005 Calculus Team Question # 4

The graph $f(x) = x^{2/3}$ is plotted on the domain $(0, a)$ where $a > 0$. The area between $f(x)$ and the *x*-axis is revolved 360° about the *y*-axis such that a solid with volume $\frac{3\pi}{64}$ is generated. Find the exact value of *a*.

Palm Harbor Invitational – February 2005 Calculus Team Question # 5

Given are 5 statements regarding the function $f(x) = e^{1/x}$
List the letters corresponding to the statements that are true regarding $f(x)$?

A $\lim_{x \to 0} f(x) = \infty$

B $f(x)$ does not have a point of inflection on the domain $(0, \infty)$

C $\lim_{x \to \infty} f'(x)$ converges

D $f(x)$ does not have any critical numbers on its domain in the interval $(-\infty, \infty)$

E $f(x)$ is never concave down and decreasing at any *x* value on its domain in the interval $(-\infty, \infty)$

Palm Harbor Invitational – February 2005 Calculus Team Question # 6

The graph of $f(x)$ is in the form $ax^3 + bx^2 + cx + d$ and it is known that the point $(2,4)$ is a point of inflection and the point $(1,-2)$ is a local minimum. Find the value of $a^2 + b^2 + c^2 + d^2$.

Palm Harbor Invitational – February 2005 Calculus Team Question # 7

Given: $g(x) = \sin(2x^2) + 1 \qquad x \in [0,4)$

A = The number of local minima of $g(x)$

B = The number of inflection points of $g(x)$

C = The number of local maxima of $g(x)$

Using the midpoint rectangle approximation, find the area under $g(x)$ on the domain $[A, B]$ with C rectangles. (Round your answer to the nearest hundredth)

Palm Harbor Invitational – February 2005 Calculus Team Question # 8

A movie theater receives an average net profit of $1.40 per customer if it allows 100 customers or less to enter daily. If more than 100 customers enter the theater, the average net profit decreases by two cents for each customer over 100. Find the number of customers that the theater should let in each day to maximize its profit.

Palm Harbor Invitational – February 2005 Calculus Team Question # 9

Find the exact minimum distance between the graphs $y = \sin(x) + x$ and $y = x - 2$.

Palm Harbor Invitational – February 2005 Calculus Team Question # 10

List A, B, C and D from highest to lowest.

$$A = \int_0^2 \frac{dx}{9 + x^2} \qquad B = \int_0^1 (x^3 + 2x^2 - 7x + 4)dx$$

$$C = \int_1^2 \left(\frac{1}{x} + 2^x\right)dx \qquad D = \int_0^3 \frac{8dx}{15 + 2x - x^2}$$

Palm Harbor Invitational – February 2005 Calculus Team Question # 11

Let l_1 be the tangent line to $y = x^3 + 2x^2 + 7x + 5$ at $x = -1$ and let l_2 be the tangent line to $y = x^3 - 3x^2 - 4x + 7$ at $x = 1$. l_1 and l_2 intersect at the point (x,y). What is $13(x + y)$?

Palm Harbor Invitational – February 2005 Calculus Team Question # 12

A box, initially with width 2 centimeters, length 3 centimeters, and height 2 centimeters, is changing in volume as a function of time. The height is constantly increasing at a rate of 0.1 mm/sec, the width is constantly increasing at a rate of 0.5 mm/sec, while the length is constantly decreasing at a rate of 0.2 mm/sec. Find the rate at which the surface area of the box is changing in cm^2/sec after 10 seconds. (Round your answer to the nearest thousandth)

Palm Harbor Invitational – February 2005 Calculus Team Question # 13

The lines tangent to the graph $y = x^2$ at $x = 1$ and $x = 2$ are drawn. Find, to the nearest degree, the smaller angle formed at the intersection of these two lines. (Round your answer to the nearest tenth of a degree)

Palm Harbor Invitational – February 2005 Calculus Team Question # 14

An isosceles triangle is changing in shape. The vertex angle, θ, and side opposite the vertex angle, y, are decreasing, while the two equal sides, x, remain constant. If the vertex angle is decreasing at a rate of 0.1° every minute, find the rate, in centimeters per minute, at which the side opposite the vertex angle is decreasing when the vertex angle is 20°, given that the triangle is initially an equilateral triangle of side length 10 cm. (Round your answer to 4 decimal places)