

**Palm Harbor Invitational Calculus Team**

1) The velocity that the amazing P@dderk drives his 1974 Pontiac is represented by the function  $v(t) = -t^3 + 40t^2 + 500t$  where  $t > 0$ .

- A) Time that the amazing p@dderk has maximum velocity
- B) P@dderk's change in position between  $t=15$  and  $t=30$
- C) Time that the amazing p@dderk has a maximum acceleration
- D) Time that the Pontiac's velocity equals zero again (after  $t=0$ )

Find  $AC+BD$  to the nearest whole number.

2) The value of the 25<sup>th</sup> derivative of  $-\cos x$ , evaluated at  $\pi/4$

The value of the 250<sup>th</sup> derivative of  $-\sin x$ , evaluated at  $\pi/2$

The value of the 2500<sup>th</sup> derivative of  $\cos x$ , evaluated at  $\pi/6$

The value of the 32136<sup>th</sup> derivative of  $\sin x$ , evaluated at  $\pi/3$

Find ABCD (exact value)

3) What are the exact values of the coordinates of the point on the graph of  $x=y^2+1$  (in the fourth quadrant) that is the minimum distance from the point (4,0)?

4) A)  $\lim_{x \rightarrow 1^+} \sqrt{x^2 - x}$

B)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x-5}}{\sqrt{x+3}}$

C) Given  $f(x) = \frac{6x^2 + 3}{2x + 7}$ , find  $f''(x)$

D)  $\int_C^B dx$

Find D (exact value)

5) A soda company makes cylindrical cans that hold 12 fluid oz. (355mL.) of fluid. If the side of the can is half as expensive as the top and bottom, and  $r$  &  $h$  are the radius and the height of the most economical can in inches, find  $r+h$  to the nearest tenth.

6) Find the centroid of the region bounded by the graphs of  $f(x) = x^2 + 6x - 7$  and  $g(x) = -$

$x^2 + 2x + 9$ , in the form of  $(x_1, y_1)$ . Evaluate  $\frac{dy}{dx}$  of  $3x^2 + 4xy + 7y^2 = 15$  at  $(x_1, y_1)$ .

7) Let region A be the region bound by  $f(x) = x^2 - 4x$ ,  $y = 12$ , and the  $y$ -axis, to the left of the  $y$ -axis.

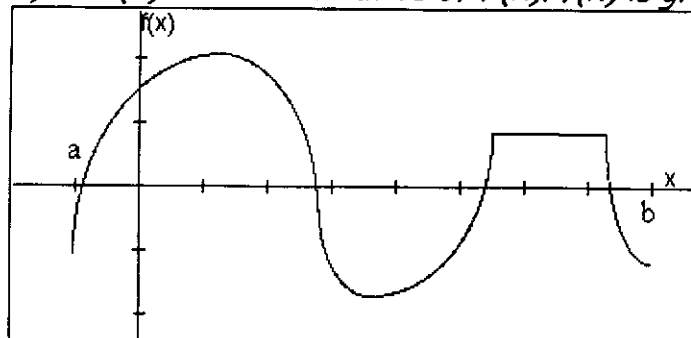
- A) Find the volume of the solid generated by revolving region A around the  $y$ -axis
- B) Find the volume of the solid generated by revolving region A around the  $x$ -axis
- C) Find the volume of the solid generated by revolving region A around the line  $y=15$
- D) Find the volume of the solid generated by revolving region A around the line  $x=5$

Find  $\frac{AB}{CD}$

8) Let  $f(x)$  be continuous and differentiable over all real numbers, where

$$f(x) = \begin{cases} \frac{5}{2}x + 7, & x < -1 \\ P(x), & -1 \leq x \leq 3 \\ 2x^2 - \frac{11}{2}x + 5, & x > 3 \end{cases}. \text{ Find } P(x), \text{ a polynomial of the smallest possible degree.}$$

9) Let  $f(x)$  be the derivative of  $F(x)$ .  $f(x)$  is graphed below.



- A) The number of extrema of  $F(x)$  on  $(a,b)$
- B) The number of inflection points on  $F(x)$  on  $(a,b)$
- C) The number of points at which  $f(x)$  is not differentiable on  $(a,b)$
- D) The smallest number of open intervals on  $(a,b)$  that contain all values of  $x$  for which  $F(x)$  is increasing.

Find ABCD

10) The first three terms of an arithmetic sequence add up to 36. A geometric sequence is formed if the second term of the arithmetic sequence is reduced by 2 and the first term is increased by 1. Find the first three terms of all arithmetic sequences that have this property.

11)  $a$ ,  $b$ , and  $c$  represent complex numbers

$$\int_1^a (\ln x) dx = 1 \quad \int_1^{\infty} \left( \frac{1}{(1+x)^b} \right) dx = \frac{1}{64} \quad \frac{c}{c+1} - \frac{1}{c+1} = c$$

Solve the following equation for  $y$ , for the smallest positive value of  $y$ :  $a^{cy} + b = 4$

12) A ball is dropped from a height of 7 feet, and begins to bounce. The rebound velocity is 75% of the velocity at impact. Find the total distance the ball travels before coming to rest. Use  $32 \text{ft/s}^2$  as the acceleration due to gravity.

13) A boat is heading straight towards the shore at a speed of 30m/s. This particular boat has a spotlight that makes one revolution every 5s. When the boat is 500m offshore, how fast, to the nearest m/s, is the beam of light moving along the shoreline if the angle the beam of light makes with the shoreline is  $\pi/3$ ?

14) The graph of  $r=1-3\cos\theta$  is a limaçon. Find the area of the closed region bounded by curves of the limaçon that contains the polar point  $(3,\pi)$ .

15) What exact value or values of  $x$  cause the series  $\sum_{n=0}^{\infty} (n+2)x^n$  to converge to

$$18 + 7\sqrt{6}?$$