PHUHS Invitational Calc Team
answers

1) 14,695,738

2) $\frac{3\sqrt{2}}{8}$

3) $\left(\frac{7}{2},-\frac{\sqrt{10}}{2}\right)$ or $(3.5,-\sqrt{2.5})$

4) $\frac{544}{441}$

5) 6.0

6) $-\frac{9}{23}$

7) $\frac{1358}{10317}$

8) $\frac{1}{2}x^3 - x^2 - x + 5$

9) 32

10) 4,12,20 and 19,12,5

11) π

12) 25

13) 855m/s

14) 12

15) $\frac{1}{3}\sqrt{6}$

Palm Harbor University Invitational
Calculus Team Solutions

1. v(t)= $-t^3+40t^2+500t \Rightarrow v'(t)=a(t)=-3t^2+80t+500$

a) max. velocity where $-3t^2+80t+500=0$ @ $\dfrac{10\left(\sqrt{31}+4\right)}{3}$

b) Change in position, area under velocity graph$\Rightarrow \displaystyle\int_{15}^{30} -t^3+40t^2+500t\,dt = \frac{1175625}{4}$

c) $a'(t)=-6t+80 \Rightarrow -6t+80=0$ @ $t=40/3$

d) $-t^3+40t^2+500t=0$ @ $t=50$ AC+BD≈ **14,695,738**

2. $\dfrac{d^{25}}{dx^{25}}[-\cos x]=\underline{\sin x} \Rightarrow$

-cos x	0	4	8...	n/4 R0
sin x	1	5	9....	n/4 R1
cos x	2	6	10...	n/4 R2
-sin x	3	7	11...	n/4 R3

25/4= 6 R1; $\dfrac{d^{250}}{dx^{250}}[-\sin x]=\underline{\sin x} \Rightarrow$

-sin x	0	4	8...	n/4 R0
-cos x	1	5	9....	n/4 R1
sin x	2	6	10...	n/4 R2
cos x	3	7	11...	n/4 R3

250/4= 62 R2;

$\dfrac{d^{2500}}{dx^{2500}}[\cos x]=\underline{\cos x} \Rightarrow$

cos x	0	4	...	n/4 R0
-sin	1	5	....	n/4 R1
-cos x	2	6	...	n/4 R2
sin x	3	7	...	n/4 R3

2500/4=625 R0; $\dfrac{d^{32136}}{dx^{232136}}[\sin x]=\underline{\sin x} \Rightarrow$

sin x	0	4	...	n/4 R0
cos x	1	5	....	n/4 R1
-sin x	2	6	...	n/4 R2
-cos x	3	7	....	n/4 R3

32136/4= 8034 R0

$\sin(\pi/4)\sin(\pi/2)\cos(\pi/6)\sin(\pi/3) = \boxed{\dfrac{3\sqrt{2}}{8}}$

3. $x=y^2+1 \Rightarrow y=\begin{cases}\sqrt{x-1}\\ -\sqrt{x-1}\end{cases}$ $d=\sqrt{(4-x)^2+\left(0--\sqrt{x-1}\right)^2}=\sqrt{x^2-7x-15} \Rightarrow$ because d is minimum when the expression inside the radical is the smallest, it is only necessary to find the critical points of the expression under the radical. $x^2-7x-15$ is minimum at $x=7/2$. ∴The minimum point in the 4th quadrant is $\boxed{\left(\dfrac{7}{2},-\dfrac{\sqrt{10}}{2}\right)}$

4. a) $\displaystyle\lim_{x\to 1^+}\sqrt{x^2-x}=\sqrt{1^2-1}=0$ c) $f(x)=\dfrac{6x^2+3}{2x+7} \Rightarrow f'(x)=\dfrac{6\left(2x^2+14x-1\right)}{(2x+7)^2}$

b) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x-5}}{\sqrt{x+3}}\cdot\frac{\sqrt{1/x}}{\sqrt{1/x}}=\lim_{x\to\infty}\frac{\sqrt{1-\frac{5}{x}}}{\sqrt{1-\frac{5}{x}}}=1 \Rightarrow$ $\displaystyle\int_0^1 f''(x)dx=\left[f'(x)\right]_0^1=\frac{6(15)}{9^2}-\frac{6(-1)}{7^2}=\boxed{\frac{544}{411}}$

5.Cost= $2\pi rh+2\times(\pi r^2)\times\dfrac{\text{cost}}{\text{square cm}} \Rightarrow$ Volume=255mL=$\pi r^2h \Rightarrow$ height in terms of radius: $\dfrac{355}{\pi r^2} \Rightarrow$

$v(r)=2\pi r\times\dfrac{355}{\pi r^2}+4\pi r^2 \Rightarrow v'(r)=\dfrac{-710}{r^2}+8\pi r$ ∴ v'(r)=0 when r=3.04559cm and h=12.182399cm. This is definitely a minimum cost since v'(3)=-3.49↓ and v'(4)=56.15596↑. There are 2.54cm/in.∴r=1.2in, and h=4.8in. (1.2+4.8= **6.0**)

6.g(x)=upper, f(x)=lower. A= $\displaystyle\int_{-4}^{2}[g(x)-f(x)]dx=72$ ∴ $x_c=\dfrac{1}{A}\displaystyle\int_{-4}^{2}[(g(x)-f(x))x]dx=-1$ &

$y_c=\dfrac{1}{A}\displaystyle\int_{-4}^{2}\left[\frac{g(x)+f(x)}{2}[g(x)-f(x)]\right]dx=-3 \Rightarrow (x_1,y_1)=\underline{(-1,-3)} \Rightarrow \dfrac{dy}{dx}=\dfrac{-6x-4y}{4x+14y}$, evaluated at (-1,-3) yields $\boxed{\dfrac{-9}{23}}$

7.a) $2\pi\displaystyle\int_0^2[x(12-(x^2-4x))]dx=\frac{56\pi}{3}$ b) $\pi\displaystyle\int_{-2}^{0}(12^2-(x^2-4x)^2)dx=\frac{3104\pi}{15}$ $\dfrac{AB}{CD}=\boxed{\dfrac{1358}{10317}}$

c) $\pi\displaystyle\int_{-2}^{0}\left[(15-(x^2-4x))^2-(15-12)^2\right]dx=\frac{2896\pi}{15}$ d) $2\pi\displaystyle\int_{-2}^{0}[(5-x)(12-(x^2-4x))]dx=152\pi$

8.The smallest possible degree is three since: f(-1)>f(3) & f'(-1)>0 & f'(3)>0, this means that there must be two turning points. $P(x)=ax^3+bx^2+cx+d$ & $P'(x)=3ax^2+2bx+c \Rightarrow P(-1)=9/2$, $P'(-1)=5/2$, $P(3)=13/2$, $P'(3)=13/2$

[-a+b-c+d=9/2 & 27a+9b+3c+d=13/2 & 27a+6b+c=13/2 & 3a-2b+c=5/2]⇒

Solving Simultaneously, a=1/2, b=-1, c=-1, d=5. ∴ P(x)= $\boxed{\frac{1}{2}x^3 - x^2 - x + 5}$

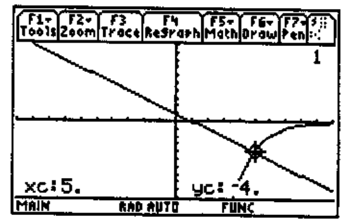
9.a) Since f(x) is the derivative of F(x), the number of extrema is equal to the number of times f(x) crosses the x-axis (4)
b) An inflection point is defined as a point where concavity changes from positive to negative(or vice versa). A point where the graph of f(x)'s slope changes from + to – (or – to +) would be a point of inflection (2)
c) f(x) is not differentiable at the sharp curves in the graph (2)
d)F(x) is increasing whenever f(x) is above the x-axis (2) ABCD= $\boxed{32}$

10. The second term of the arithmetic sequence is the average of the three 36/3=12. Then the first term is 12-d, and the third is 12+d. The first term of the geometric sequence is 13-d, and the second term of the geometric sequence is 10. Therefore, $\frac{10}{13-d} = \frac{12+d}{10} \Rightarrow$ d= 8, -7. ∴ The two arithmetic series are $\boxed{\text{4, 12, 20 and 19, 12, 5}}$

11. $\int_1^a (\ln x)dx = [x\ln x - x]_1^a = 1 \Rightarrow$ (a ln a –a)-(1 ln 1 –1)=1⇒a ln a-a=0⇒a=e

$$\int_1^\infty \left(\frac{1}{(1+x)^b}\right)dx = \frac{1}{64} \Rightarrow \left[\frac{(1+x)^{-b+1}}{-b+1}\right]_1^\infty = \frac{1}{64} \Rightarrow \frac{(\infty+1)^{-b+1}}{-b+1} - \frac{2^{-b+1}}{-b+1} = \frac{1}{64} \Rightarrow 2^6 \times -\frac{2^{-b+1}}{-b+1} = \frac{1}{64} \times 2^6 \Rightarrow -2^{-b+7} = -b+1$$

Graphing the equation, since it is not readily solvable, the graphs intersect at 5.



$\frac{c-1}{c+1} = c \Rightarrow c^2+c=c-1 \Rightarrow c^2+1=0 \Rightarrow c=\pm i$ $\quad a^{cy} + b = 4 \Rightarrow e^{iy} + 5 = 4$ ∴ y= $\boxed{\pi}$

12. $a(t)=-32 \Rightarrow v(t)=-32t+C_1 \Rightarrow s(t)=-16t^2+C_1t+C_2 \Rightarrow C_2=7 \Rightarrow v(0)=0 \therefore C_1=0$
$s(t)=-16t^2+7 \Rightarrow s(t)=0$ when $7=16t^2$ or $t=\sqrt{7}/4 \Rightarrow v(\sqrt{7}/4)=-32+0 \Rightarrow$
$-32(\sqrt{7}/4)=-8\sqrt{7} \Rightarrow -8\sqrt{7}\,(3/4)=6\sqrt{7} \Rightarrow s(t)=-16t^2+6\sqrt{7}+0 \Rightarrow t=3\sqrt{7}/16$

$s(3\sqrt{7}/16)=63/16 \Rightarrow a_n=7+63/16,\ldots \therefore (63/16)/7=9/16 \therefore \sum_{n=0}^{\infty}\left(\frac{9}{16}\right)^n 14$ & $\left|\frac{9}{16}\right| < 1 \therefore \frac{14}{1-(9/16)} - 7 \therefore \frac{14}{\frac{16-9}{16}} = 32$

Total distance: $\sum_{n=0}^{\infty} 14\left(\frac{9}{16}\right)^n = 32 \Rightarrow$ 32-7= $\boxed{25}$

13.Given dy/dt=-30, y=500, θ=π/6(since the other angle is π/3) dθ/dt=-2π/5⇒ tan θ=x/y⇒ x=y tanθ
$dx/dt=\tan\theta\, dy/dt\, \sec^2\theta\, d\theta/dt=\tan(\pi/6)\text{-}30+500\sec^2(\pi/6)(-2\pi/5)=1/3\sqrt{3}(-30)+500(4/3)(-2\pi/5)\approx -855$ ∴ 855m/s

14.r=1-3cosθ A= $\int_a^b \frac{1}{2}r^2 \theta d\theta \Rightarrow \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(1-3\cos\theta)^2 d\theta = \frac{11\pi+24}{4}$ (area inside the outer loop)

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1-3\cos\theta)^2 d\theta = \frac{11\pi-24}{4}$ (area inside the inside loop) ∴ Area= $\frac{11\pi+24}{4} - \frac{11\pi-24}{4} = \boxed{12}$

15. $\sum_{n=0}^{\infty}(n+2)x^n = 2+3x+4x^2+5x^3\ldots = (1+x+x^2+x^3\ldots)+(1+2x+3x^2+4x^3\ldots) = \frac{1}{1-x} + \frac{1}{(1-x)^2} \Rightarrow$

$\frac{1}{1-x} + \frac{1}{(1-x)^2} = 18+7\sqrt{6} \Rightarrow (1-x)+1 = (18+7\sqrt{6})(1-x)^2 = (18+7\sqrt{6})(x^2-2x+1) \Rightarrow$

0= $(18+7\sqrt{6})x^2 + (-35-14\sqrt{6})x + (16+7\sqrt{6}) \Rightarrow x = \frac{14-\sqrt{6}}{10}$ or $\frac{\sqrt{6}}{3} \Rightarrow$ Since $\left|\frac{14-\sqrt{6}}{10}\right| > 1$ the series does not converge for that value ∴ x= $\boxed{\frac{\sqrt{6}}{3}}$