February 2005 Palm Harbor Invitational Calculus Individual Test

Let choice E) NOTA denote "None Of These Answers."

1) What is the y-coordinate of the point of inflection of the graph $y = \sqrt{1 - \ln x}$ over the domain $(0, e)$?

A) 0 B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{e}$ E) NOTA

2) Solve the differential equation, $\frac{dy}{dx} = x + \frac{3x}{\sqrt{25 - x^2}}$, satisfying the condition $y(0) = -10$.

A) $y = x^3 + 3\sqrt{25 - x^2} - 26$ B) $y = x^3 - 3\sqrt{25 - x^2} + 5$

C) $y = \frac{x^2}{2} + 3\sqrt{25 - x^2} - 25$ D) $y = \frac{x^2}{2} - 3\sqrt{25 - x^2} + 5$ E) NOTA

3) Evaluate: $\int_0^2 \left(\frac{1}{x-1}\right)^{2/3} dx$

A) $\frac{6}{5}$ B) 6 C) ∞ D) Does not exist E) NOTA

4) $y = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}}}$, $x, y > 0$ Find $\frac{dy}{dx}$ at the point where $x = 2$.

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) 1 E) NOTA

5) Let $f(x)$ and $g(x)$ be continuous and differentiable functions on the domain of all real values. $\lim_{x \to 0} \frac{g(x)}{f(x)}$ diverges. Which of the following statements MUST be true?

I. $g(0) \neq 0$.

II. $\lim_{x \to 0} \frac{f(x)}{g(x)}$ will not diverge.

III. $f(0) = 0$.

IV. $\lim_{x \to 0} \frac{g'(x)}{f'(x)}$ diverges.

A) I only B) III. only

C) II & III only D) I, III, & IV only E) NOTA

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6) Using the left-hand Riemman sum approximation, calculate the area under the graph $f(x) = x^2$ with 2005 sub-divisions. (Round your answer to 4 decimal places)

A) 0.3330 B) 0.3331 C) 0.3333 D) 0.3336 E) NOTA

7) Find $A + B$ such that the following limit is true:
$\lim_{x\to\infty}\left[\frac{2x^2-7}{x+3} - (Ax+B)\right] = 0$.

A) –4 B) –2 C) 2 D) 8 E) NOTA

8) Evaluate: $\int 3^x\,dx$

A) $3^x \ln 3 + C$ B) $\frac{3^x}{\ln 3} + C$ C) $e^{x\ln 3} + C$ D) $Ce^{x\ln 3}$ E) NOTA

9) Evaluate the limit: $\lim_{x\to 0}\frac{3x^4 - 3x^3 + 5x^6 + 4x - 1}{2x^4 + 7x^3 - 6x^6 + 2 - x^2}$

A) $-\frac{5}{6}$ B) $-\frac{3}{2}$ C) $-\frac{3}{7}$ D) $\frac{3}{2}$ E) NOTA

10) The line $2y = x + 4$ divides the ellipse $x^2 + 4y^2 = 16$ into two regions, A_1 and A_2 where $A_1 < A_2$. Find $\frac{A_1}{A_2}$ to 2 decimal places.

A) 0.10 B) 0.12 C) 0.15 D) 0.20 E) NOTA

11) Given: $h(x) = e^{\sqrt{\sin x}}$ Find: $h''\left(\frac{\pi}{2}\right)$

A) $\frac{e}{4}$ B) 0 C) $-\frac{e}{2}$ D) Does not exist E) NOTA

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12) The position of a particle moving along the *x*-axis is defined by the following function: $x(t)=\frac{1}{2}\left(e^{2t}-10e^{t}+12t\right)$ where $x(t)$ is the *x*-coordinate of the particle on the *x*-axis at time *t*, where $t \geq 0$. Find the acceleration of the particle at the first instance when its velocity is equal to zero.

A) $2e^4-5e^2$ B) 3 C) 0 D) -2 E) NOTA

13) What is the equation of the tangent line to $y=\cos 2x-\sin x$ at $x=\pi$?

A) $x+y=\pi+1$ B) $x-y=1-\pi$
C) $x+y=\pi-1$ D) $x-y=\pi-1$ E) NOTA

14) Evaluate the following improper integral: $\int_0^{\infty}\frac{dx}{x^2+a^2}$, $a \in \Re$, $a>0$

A) $\frac{\pi}{4a}$ B) $\frac{\pi}{2a}$ C) $\frac{\pi}{a^2}$ D) $\frac{\pi}{2a^2}$ E) NOTA

15) Given: $g(x)=\tan\left(x^2\right)+\tan^2\left(x^2\right)$ Find $g'\left(\sqrt{\pi}\right)$

A) $-4\sqrt{\pi}$ B) 0 C) $2\sqrt{\pi}$ D) $6\sqrt{\pi}$ E) NOTA

16) Find the sum of the values for *c* that satisfy the Mean Value Theorem for Derivatives of the function $f(x)=x^3-x$ over the domain $[-2,1]$.

A) -1 B) 0 C) 1 D) 2 E) NOTA

17) Given: $f(x)=\ln\left(x^3+2\right)$. Find: $\frac{d}{dx}\left[f^{-1}(x)\right]$ when $x=0$.

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{\ln(2)}$ D) Does not exist E) NOTA

18) Given: $g(x)=\begin{cases}\ln(x+1)/x & x\neq 0\\ 1 & x=0\end{cases}$ Find: $g'(0)$.

A) Not possible B) $\frac{1}{4}$ C) 0 D) $-\frac{1}{2}$ E) NOTA

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19) Find the area of the largest rectangle that can be inscribed between the graph $y = 2x - x^2$ and the *x*-axis where one side of the rectangle is lying on the *x*-axis. (Round your answer to the nearest hundredth)

A) 0.77 B) 0.89 C) 1.19 D) 1.31 E) NOTA

20) The line tangent to the graph $f(x) = (x-2)^3(x^2+2)^2$ at $x = 1$ intersects the *x*-axis at $(a,0)$. Find the tenth's digit of $\ln|a|$.

A) 1 B) 2 C) 4 D) 6 E) NOTA

21) Using Newton's Method, calculate x_1 in approximating the zero to $y = 2^x - x$ with an initial guess of $x_0 = 1$. (Round your answer to the nearest hundredth)

A) –1.59 B) 0.39 C) 0.61 D) 2.59 E) NOTA

22) A particle's position on the *x*-axis is defined by the function $x(t) = t^3 - 3t^2 - 9t + 3$, where *t* is time in seconds. What is the total distance traveled by the particle between $t = 0$ and $t = 4$?

A) 7 B) 20 C) 27 D) 34 E) NOTA

23) A 3-dimensional solid's base in the *xy*-plane is the region bound by the graph of $y = 1 - x^2$ and the *x*-axis. Its cross sections perpendicular to the *y*-axis are squares. What is the volume of the solid?

A) 2 B) $\frac{16}{15}$ C) $\frac{8}{15}$ D) $\frac{1}{2}$ E) NOTA

24) Find the slope of $x^3y + x^2y^2 + xy^3 + 1 = 0$ at $(1,-1)$.

A) –1 B) 0 C) 1 D) 2 E) NOTA

25) Let $f(x) = \sqrt[3]{x^2} - x$. Which of the following statements are true?

I. $f(x)$ is differentiable at every real value of *x*.

II. $f(x)$ is always decreasing.

III. $f(x)$ has a point of inflection at $(0,0)$.

IV. $f'(x)$ is an odd function.

A) III. only B) II., & III. only

C) II. & IV. only D) I., III., & IV. only E) NOTA

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26) Evaluate: $\int_0^{\frac{\pi}{4}} \sin^2 x\,dx$

A) 0 B) $\frac{\pi-2}{8}$ C) $\frac{\pi}{4}$ D) $\frac{\pi-4}{2}$ E) NOTA

27) The pressure of a given gas in a closed chamber varies with its volume given by $P=\frac{C}{V}$ where C is a constant. Initially the pressure of the gas is $1.4\frac{N}{m^2}$ at a volume of $0.25\,m^3$. The gas then expands to $0.96\,m^3$ by doing work on the chamber. Find the amount of work in joules the gas does on the chamber while expanding rounded to two decimal places.

A) 0.25 B) 0.47 C) 1.04 D) 1.35 E) NOTA

28) Let $f(x)=e^{-2x}$. What is a formula for the nth derivative of $f(x)$?

A) $(-2)^n e^{-2x}$ B) $(-1)^n e^{-2x}$
C) $(-1)^{n-1}2^n e^{-2x}$ D) e^{-2x} E) NOTA

29) What is the area of the region bounded by the curves $y=2x^2+5x-6$ and $y=x^2+2x+4$?

A) $\frac{35}{6}$ B) $\frac{91}{2}$ C) $\frac{343}{6}$ D) $\frac{383}{2}$ E) NOTA

30) Evaluate: $\frac{d}{dx}\left[\frac{x}{\sqrt{x}}\right]$ at $x=4$.

A) $-\frac{1}{4}$ B) 0 C) $\frac{1}{4}$ D) 2 E) NOTA