2004 Palm Harbor Invitational Calculus Individual Test

Let choice E) NOTA denote "None Of These Answers"

1) Find the maximum value of $g(x) = x^3 + 5x^2 - 8x + 2$ on the interval $[-5,4]$.

A) 42 B) 50 C) 114 D) 212 E) NOTA

2) Which of the following functions have a non-differentiable cusp at $x = 0$?

$f(x) = x^{4/7}$ $g(x) = \ln(x^2)$ $h(x) = e^{1/x^2}$ $j(x) = x^{1/3} \cdot (2x-1)^{2/3}$

A) f only B) f & j only

C) g & h only D) g, h, & j only E) NOTA

3) Evaluate: $\lim_{x \to 0} \frac{1}{x} \int_0^x (t^2 - t + 2)\,dt$

A) 0 B) 1 C) 2 D) Does Not Exist E) NOTA

4) A rectangle is inscribed in the first quadrant with two edges lying on the x- and y-axes and a vertex lying on the graph of $h(x) = 2^{1-x^2}$. When the rectangle is at its maximum area, the vertex on the graph of $h(x)$ has coordinate (a,b). To the nearest hundredth, calculate the value of $a + b$.

A) 1.92 B) 2.00 C) 2.06 D) 2.12 E) NOTA

5) What is the sum of the values that satisfy the Mean Value Theorem for the function $y = x^4 - 8x^3 + 22x^2 - 15x + 3$ on the interval $[1,3]$?

A) 2 B) 6 C) 11 D) 24 E) NOTA

6) Evaluate: $\int_0^{\frac{\pi}{4}} \sec^2 x \tan x\,dx$

A) $-\frac{3}{2}$ B) $\frac{1}{2}$ C) 1 D) $\frac{3}{2}$ E) NOTA

7) Evaluate: $\lim_{x \to 0} \frac{\sin(2x) - 2\sin x}{x^2}$

A) 0 B) 1 C) 2 D) Does Not Exist E) NOTA

8) Let $y = ax^3 + bx^2 - 2x + 1$. If the point (1,1) is a point of inflection of the given function, what is the value of $a^2 + b^2$?

A) 2 B) 5 C) 10 D) 13 E) NOTA

9) Given: $f(x) = x^3 + 2x + 5$ and $f^{-1}(x) = g(x)$

Find: $g'(2)$

A) $-\frac{14}{289}$ B) $-\frac{1}{2}$ C) $\frac{1}{14}$ D) $\frac{1}{5}$ E) NOTA

10) The graph of $y = \frac{1}{2}\sqrt{1-4x^2}$ on the domain of $\left[0, \frac{1}{2}\right]$ is revolved around the y-axis. What is the lateral surface area of the region formed?

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) π D) 2π E) NOTA

11) Given: $f(x) = ax^4 + bx$; $a, b \in \Re; a, b \neq 0$

Which of the following MUST be true?

I. $f(x)$ has an inflection point at $x = 0$

II. The graph of $f''(x)$ is either always concave up or concave down

III. $|f(1)| < |f'(1)| < |f''(1)|$

IV. The graph of $f'(x)$ will not have more than one inflection point

A) II. only B) II. & IV. only

C) I., II., & IV. only D) II., III., & IV. only E) NOTA

12) Find the equation of the normal line to the graph of $xy^2 - 3x^2y = 4$ at $x = 1$ in the fourth quadrant.

A) $5x + 7y = -2$ B) $5x - 7y = 12$

C) $7x + 5y = 2$ D) $7x - 5y = 12$ E) NOTA

13) Find the minimum distance between the graph of $f(x) = x^2 - 8$ and the point $(0,5)$.

A) $\frac{1}{2}$ B) $\frac{3\sqrt{2}}{2}$ C) $\frac{5\sqrt{2}}{2}$ D) $\frac{\sqrt{51}}{2}$ E) NOTA

14) The temperature of a cup of coffee follows the cooling pattern established by Newton's Law of Cooling. If the initial temperature is 95.0 °C and after 2 minutes it is 65.0 °C, then how long will it take to reach 40.0 °C if the surrounding temperature is 27.0 °C? (Round your answer to the nearest tenth of a minute)

A) 2.5 B) 3.3 C) 4.1 D) 4.9 E) NOTA

15) Evaluate the limit: $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i^6}{n^7}\right)$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{7}$ D) ∞ E) NOTA

16) Use differentials to approximate $\ln(e-1)$ given the fact that $\ln(e)=1$. (Round your answer to the nearest thousandth)

A) 0.368 B) 0.418 C) 0.541 D) 0.632 E) NOTA

17) Which of the following functions meets all the criteria of Rolle's Theorem on the interval $[-4, 4]$?

A) $f(x)=\frac{3}{2}x^{\frac{2}{3}}$ B) $f(x)=\frac{x^4-9}{x^2-3}$ C) $f(x)=|x+5|$

D) $f(x)=\left|\cos\left(\frac{\pi x}{8}\right)\right|$ E) NOTA

18) What is the average value of $y = \ln x$ from $x = 2$ to $x = 4$?

A) $\ln\left(\frac{27}{4}\right)-1$ B) $\frac{3}{2}\ln 2$ C) $3\ln 2-1$ D) $6\ln 2-2$ E) NOTA

19) The region bounded by the graphs of $f(x)=x+1$ and $g(x)=x^2+1$ is revolved 360° about the y-axis. Find the volume of the solid generated.

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{7\pi}{15}$ D) $\frac{3\pi}{5}$ E) NOTA

20) Evaluate: $\lim_{x\to\infty}\frac{4x^2+2x+3x^3}{5x^3+4x^2+x+1}$

A) 0 B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) 1 E) NOTA

21) Find the 2004^{th} derivative of $\text{cis}(\pi x)$ where $\text{cis}A = \cos A + i\sin A$ and i is treated as a constant.

A) $\text{cis}(\pi x)$

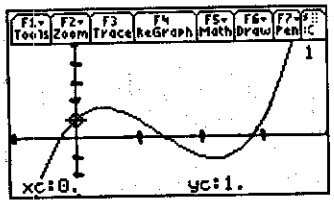
B) $\pi^{2004}\text{cis}(x)$

C) $\pi^{2004}e^{i\pi x}$

D) $-\pi^{2004}\text{cis}(\pi x)$

E) NOTA

22) The following graph represents a portion of the graph of $f'(x)$ on the domain $[-1,4]$ and the range $[-3,5]$. (Note: each tick mark represents one unit)



Which of the following MUST be true?

A) $f'(x)$ has a local minimum between $x = 1$ and $x = 2$

B) $f(x)$ has a root between $x = 0$ and $x = 1$

C) $f(x)$ is concave down between $x = 1$ and $x = 2$

D) $f''(x)$ has a local maximum between $x = 1$ and $x = 2$

E) NOTA

23) Find the volume of a solid whose base is bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with cross-sections of equilateral triangles perpendicular to the y-axis.

A) $30\sqrt{3}$ B) $50\sqrt{3}$ C) $60\sqrt{3}$ D) $100\sqrt{3}$ E) NOTA

24) Find the particular solution to the given differential equation: $x^2y' - y^2 = 4$, satisfying the initial condition $y(1) = 0$.

A) $\arctan\left(\frac{y}{2}\right) = 2 - \frac{2}{x}$

B) $\arctan y = 1 - \frac{1}{x}$

C) $\arctan\left(\frac{y}{2}\right) = 2\arctan x - \frac{\pi}{2}$

D) $\ln\left|\frac{y+2}{y-2}\right| = 1 - \frac{1}{x}$

E) NOTA

25) An 8 foot tall robot is walking at a constant speed of 4 feet per second away from a light pole that is 20 feet tall. At what rate, in feet per second, is the tip of his shadow moving away from the bottom of the light pole?

A) $\frac{5}{2}$ B) $\frac{8}{3}$ C) $\frac{11}{3}$ D) 4 E) NOTA

26) If $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ then what is $\sum_{n=1}^{\infty} \frac{(x-1)^n(-1-x)^n}{n!}$?

A) $e^{x^2}-1$ B) e^{-x^2-1} C) e^{1-x^2} D) $e^{1-x^2}-1$ E) NOTA

27) Evaluate: $\int \frac{2dx}{x\sqrt{x^4-1}}$

A) $\sec^{-1} x + C$ B) $\tan^{-1}(x^4-1)+C$

C) $\frac{1}{2}\ln(x^4-1)+C$ D) $\sec^{-1}(x^2)+C$ E) NOTA

28) Using the Newton-Raphson method, calculate x_2 in approximating the zero for $y = \sin x - x^2$ with an initial guess of $x_0 = 4$. (Round your answer to the nearest thousandth)

A) 0.877 B) 1.329 C) 2.064 D) 2.849 E) NOTA

29) A box is created by cutting four congruent squares from a square sheet of paper with side length 13.2 and folding up the edges. If the maximum value of the volume of the box is represented by a, then what is the tenth's digit of $\ln a$?

A) 1 B) 3 C) 7 D) 8 E) NOTA

30) A cone plotted in a three-dimensional graph is oriented in such a way that it has a constant volume of 6π cubic inches but both the height and radius are changing at constant rates. At the instant when the height of the cone is 2 inches, the radius is increasing at a rate of 3 inches/minute. In inches/minute, by how much is the height decreasing at this instant?

A) $\frac{4}{3}$ B) 4 C) 12 D) 36 E) NOTA