2003 Palm Harbor Invitational
Calculus Individual Test

Let choice "E. NOTA" denote "None of These Answers"
Assume that all numbers and solutions are over the set of real numbers.

1. Find the area of the region enclosed by $f(x)=x^2+6x-1$ and $g(x)=x-7$.

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. 1 E. NOTA

2. $\lim_{x\to 2}\frac{x^3+x^2+ax-2}{x^2-x-2}=b$ where a and b are real numbers. Find $a+b$.

A. 0 B. $\frac{7}{3}$ C. $-\frac{4}{3}$ D. Does not exist E. NOTA

3. Find the minimum value of $y=x^4-4x^3-20x^2+96x+112$ on the interval $[-2,5]$.

A. −167 B. −112 C. 176 D. 217 E. NOTA

4. $\int_a^b (a-x)(x-b)dx=12$ where a and b are real numbers such that $a<b$. Find $b-a$.

A. 3 B. 4 C. $2\sqrt[3]{9}$ D. $6\sqrt{2}$ E. NOTA

5. Use differentials to approximate $\sin 40°$ using $\sin 45°=\frac{\sqrt{2}}{2}$. Round your answer to the nearest thousandth.

A. 0.643 B. 0.644 C. 0.645 D. 0.646 E. NOTA

6. For what values of x is $f(x)=\frac{x^2+3x-2}{x+1}$ concave up?

A. $(-1,1)$ B. $(-\infty,-1)$ C. $(-1,\infty)$ D. $(-\infty,\infty)$ E. NOTA

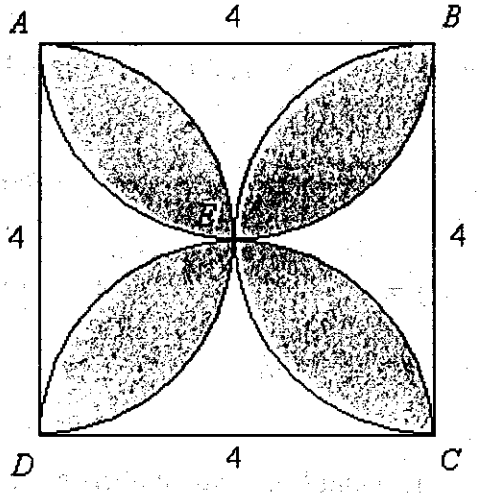
7. What is the eccentricity of a circle?

A. −1 B. 0 C. 1 D. π E. NOTA

8. Evaluate: $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$.

A. $\sqrt{2}$ B. $\frac{3}{2}$ C. $\frac{\pi^2}{6}$ D. $\sqrt{e}$ E. NOTA

9. Let quadrilateral *ABCD* be a square with sides of length 4. Four semicircles of radius 2 are introduced as shown below, such that they all intersect at point *E*. What is the area of the shaded region?



NOTE: Figure is NOT drawn to scale

A. $4-\pi$ B. $16-4\pi$ C. $32-8\pi$ D. $8\pi-16$ E. NOTA

10. The value of $\int_0^7 \frac{x}{\sqrt{x^2+1}}dx$ is simplified and written in the form $a\sqrt{b}+c$. Find $a+b+c$.

A. 6 B. 7 C. 8 D. 49 E. NOTA

11. The diagonal of a cube increases at a rate of 5 feet per second. At what rate, in cubic feet per second, is the volume of the cube increasing when the length of one side of the cube is 2 feet?

A. $20\sqrt{2}$ B. $20\sqrt{3}$ C. $30\sqrt{2}$ D. $40\sqrt{3}$ E. NOTA

12. Find the volume of the solid whose base is the circle $x^2+y^2=9$ and every cross-section perpendicular to the *x*-axis is a semi-circle.

A. $\frac{9\pi^2}{4}$ B. $\frac{9\pi}{2}$ C. 9π D. 18π E. NOTA

13. The price elasticity of demand, which measures consumers' responsiveness to changes in price, is given by $E_d = \frac{p/x}{dp/dx}$, where p is the demand function and x is the quantity sold. A product is unit elastic if E_d is equal to 1. If the demand function for a product is given by $p = 2500x - .01x^2 - 1849$, find the quantity that must be sold for the product to be unit elastic (to the nearest whole number).

A. 430 B. 1526 C. 125,000 D. 250,002 E. NOTA

14. Find the area of the fourth quadrant region bound by $f(x) = x^3 - 2x^2 - 5x + 6$ and the x-axis.

A. $-\frac{16}{3}$ B. $-\frac{37}{12}$ C. $\frac{37}{12}$ D. $\frac{16}{3}$ E. NOTA

15. Find the coefficient of the x^{64} term of the Maclaurin series expansion of $\sin x$.

A. -2 B. $-\frac{1}{64!}$ C. $\frac{1}{64!}$ D. 2 E. NOTA

16. Which of the following is equal to $\int_a^b f(x)dx$?

A. $\int_{a+c}^{b+c} [f(x)+c]dx$ B. $\int_{a+c}^{b+c} f(cx)dx$

C. $\int_{a+c}^{b+c} f(x+c)dx$ D. $\int_{a+c}^{b+c} f(x-c)dx$ E. NOTA

17. Evaluate: $\int_1^2 x^{2x}(1+\ln x)dx$

A. 3 B. $\frac{15}{2}$ C. 15 D. 16 E. NOTA

18. The Acme Box Company produces uncovered boxes by cutting congruent squares out of each corner of a rectangular piece of cardboard, and then folding up the sides. If the company uses a piece of cardboard with sides of length 10 inches and 12 inches, then what is the maximum volume of the box? Express your answer in cubic inches rounded to the nearest hundredth.

A. 5.51 B. 72.00 C. 87.50 D. 96.77 E. NOTA

19. The rate of change of the concentration, $[A]$, in a first order chemical reaction is given by $\frac{d[A]}{dt} = -k[A]$ where k is the equilibrium constant and t is the time in seconds. If $k = 1.32\times10^{-5}s^{-1}$, find the half-life of the reaction. Round your answer to the nearest hundredth of an hour.

A. 3.67 B. 6.33 C. 14.59 D. Not enough Information E. NOTA

20. Find the sum of the squares of all real values of a such that $\lim_{x\to a}\frac{x^3-7x-6}{x+1} = 14$.

A. 41 B. 42 C. 55 D. 56 E. NOTA

21. Find the n^{th} derivative of $f(x) = \ln x^2$, $n \in N$.

A. $(-1)^{n+1}(n-1)!x^{-n}$ B. $(-1)^{n+1}2(n-1)!x^{-n-1}$
C. $(-1)^{n+1}(n-1)!x^{-n-1}$ D. $(-1)^{n+1}2(n-1)!x^{-n}$ E. NOTA

22. Let $ABCD$ be a convex quadrilateral where diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E such that $m\angle AED = 45°$. If the length of $\overline{AC}$ is $10-x$ and the length of $\overline{BD}$ is $2x-4$, what is the maximum area of the quadrilateral?

A. 6 B. $8\sqrt{2}$ C. $\frac{128\sqrt{2}}{9}$ D. $16\sqrt{2}$ E. NOTA

23. Find the value of c guaranteed by the First Mean Value Theorem for Integrals for the function $f(x) = x^3 - 8$ on the interval $[2,4]$.

A. $\frac{2\sqrt{21}}{3}$ B. $\sqrt[3]{30}$ C. 22 D. 28 E. NOTA

24. Let $f(x)=4x^3-6x^2+3x-12$. Find the sum of all real values of a such that $f'(a)=f''(a)$.

A. $\frac{1}{2}$ B. $\frac{3}{2}$ C. 6 D. 36 E. NOTA

25. Use the trapezoidal rule to approximate $\int_0^1 \frac{4}{1+x^2}\,dx$ at $n=5$. Round your answer to the nearest thousandth.

A. 3.135 B. 3.141 C. 3.147 D. Trapezoidal rule cannot be used E. NOTA

26. Let $f(x)=\left|\frac{x^2-31x+58}{x-2}\right|$, $3\le x\le 29$. Which of the following statements is true?

A. The Mean Value Theorem for Derivatives can be applied
B. The Mean Value Theorem for Derivatives cannot be applied because $f(3)\neq f(29)$
C. The Mean Value Theorem for Derivatives cannot be applied because $f(x)$ is not differentiable when $x=29$
D. The Mean Value Theorem for Derivatives cannot be applied because $f(x)$ is not continuous at $x=2$.
E. NOTA

27. Newton's Law of Cooling states that the rate at which an objects cools is proportional to the difference of the object's current temperature and the temperature of its surroundings. Hot coffee is poured into a Styrofoam cup. The initial temperature of the coffee is $90°$, and the coffee cools to $60°$ in 5 minutes. Find the temperature of the coffee after 10 minutes if the temperature of the coffee's surroundings remains constant at $20°$. Round your answer to the nearest degree.

A. $30°$ B. $43°$ C. $45°$ D. $52°$ E. NOTA

28. Which of the following is equivalent to $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\left(\frac{4i}{n}\right)^2\right)^5\left(\frac{2}{n}\right)$?

A. $\int_0^1 (1+16x^2)^5\,dx$ B. $\int_0^2 (1+4x^2)^5\,dx$

C. $\int_0^4 (1+x^2)^5\,dx$ D. $\int_1^2 1024x^{10}dx$ E. NOTA

29. Evaluate: $\int \arcsin x\,dx$

A. $\frac{1}{\sqrt{1-x^2}}+C$ B. $-\arccos x+C$

C. $x\arcsin x+\sqrt{1-x^2}+C$ D. $\frac{\sqrt{1-x^2}}{\arcsin x}+C$ E. NOTA

30. Let $g(x)=\int_0^x (4t-2)dt$ and $f(x)=[g(x)]^2+g(3+x)-2$. Find $f'(2)$.

A. 26 B. 54 C. 66 D. 72 E. NOTA