Calculus Individual Test

Let choice "E) NOTA" denote "None of the above"
Assume that all numbers and solutions are over the set of real numbers.

1. Evaluate: $\lim_{x\to 2} \dfrac{x^2+x-6}{x^3-4x^2+x+6}$

A) $-\dfrac{5}{3}$ B) 0 C) $\dfrac{1}{2}$ D) $\dfrac{4}{5}$ E) NOTA

2. The Marathon Toy Company produces dolls at a rate of $f(t)=t^2-\sqrt{t}$ per hour, where t is measured in hours. How many dolls are produced during the first 9 hours?

A) 78 B) 153 C) 225 D) 287 E) NOTA

3. Find the area of the first-quadrant region bounded by $y=\dfrac{1}{x^2}$, $y=0$ and to the right of $x=1$.

A) $\dfrac{1}{2}$ B) $\ln 2$ C) 1 D) 2 E) NOTA

4. Use differentials to approximate $\sqrt[5]{250}$ to the nearest ten-thousandth given that $\sqrt[5]{243}=3$.

A) 3.0170 B) 3.0171 C) 3.0172 D) 3.0173 E) NOTA

5. If $\int_5^a \dfrac{2x-5}{x^2-5x+8}\,dx=-\ln 4$, find the sum of all real values of a.

A) -5 B) 5 C) 10 D) 11 E) NOTA

6. Find the slope of the line tangent to $f(x)=\dfrac{x^3+2x^2-x+5}{x+3}$ at $x=2$.

A) 3.04 B) 5.96 C) 6.41 D) 9.37 E) NOTA

7. Which of the following are true, assuming $C(x)$ is the cost function, $R(x)$ is the revenue function, and x is the quantity produced?

I. The marginal cost function is equal to the derivative of the cost function

II. The marginal average cost function is equal to $\frac{C'(x)}{x}$

III. The total revenue function is equal to $xp(x)$, where $p(x)$ is the demand function.

IV. The profit function is equal to $R(x)-C(x)$

A) I and II only B) I and III only C) I, II, and III only D) I, II, III, and IV E) NOTA

8. Solve the differential equation $\frac{dy}{dx}=1+3x^2+y^2+3x^2y^2$.

A) $y=x+2x^3$ B) $y=\pm\sqrt{e^{x^3+x}-1+C}$

C) $y=\tan\left(x^3+x+C\right)$ D) $y=\cot x-x^3-3x$ E) NOTA

9. Evaluate: $\int \tan x \sec^3 x \, dx$

A) $\sec^3 x+C$ B) $\sec x \tan^2 x+C$

C) $\sec x \tan x+C$ D) $\frac{\sec^3 x}{3}+C$ E) NOTA

10. Which of the following statements are true about the Mean Value Theorem for a function $f(x)$ on the interval $[a,b]$?

I. $f(x)$ must be continuous on $[a,b]$

II. $f(x)$ must be differentiable on $[a,b]$

III. There exists a number c in (a,b) such that $f(b)-f(a)=f'(c)(b-a)$

A) I and II only B) I and III only C) II and III only D) I, II, and III E) NOTA

11. Let $\int_{-4}^{6} f(x)\,dx = m$ and $\int_{6}^{11} f(x)\,dx = n$. If $\int_{11}^{-4} [f(x)+4]\,dx$ is written in the form $Am + Bn + C$ where A, B, and C are real numbers, find $A + B + C$ given that $f(x)$ is continuous on $[-4, 11]$.

A) -62 B) -30 C) 30 D) 62 E) NOTA

12. $f''(x) = 6x + 8$. If $f(2) = 21$ and $f(1) = 14$, find $f(3)$.

A) 7 B) 28 C) 36 D) 48 E) NOTA

13. Evaluate $f^n(x)$ at $x = n$ where $f(x) = xe^{-x}$ and n is a natural number.

A) 0 B) e^{-n} C) 1 D) n E) NOTA

14. Lindsay is blowing up a spherical balloon. The volume is increasing at a rate of 3 units^3 per second. At what rate is the diameter of the balloon increasing when the radius is 2 units? All answers are in units per second.

A) $\frac{3}{32\pi}$ B) $\frac{3}{16\pi}$ C) $\frac{3}{8\pi}$ D) $\frac{9}{4\pi}$ E) NOTA

15. Evaluate: $\int_{0}^{\pi} x^2 \cos x\,dx$

A) π B) 2π C) π^2 D) $2\pi^2$ E) NOTA

16. A box has sides with lengths x, $x+2$, and $5-x$. What is the maximum volume of the box? Round your answer to the nearest thousandth.

A) 1.082 B) 3.082 C) 6.041 D) 30.041 E) NOTA

17. Evaluate: $\int e^x \cos x \, dx$

A) $\frac{e^x(\cos x+\sin x)}{2}+C$ B) $e^x(\cos x+\sin x)+C$

C) $\frac{e^x(\cos x-\sin x)}{2}+C$ D) $e^x(\cos x-\sin x)+C$ E) NOTA

18. What is the slope of the line tangent to the curve defined by the parametric equations $x=\cos t$ and $y=t^2+2t-4$ at $t=\frac{\pi}{2}$?

A) $-\pi-2$ B) $-\pi+2$ C) $\pi-2$ D) $\pi+2$ E) NOTA

19. Evaluate the derivative of $y=x^3+4x^2-4x+2$ with respect to $\ln x$.

A) $3x^3+8x^2-4x$ B) $\frac{3[\ln x]^2+8\ln x-4}{x}$

C) $3[\ln x]^2+8\ln x-4$ D) $3x^2+8x-4$ E) NOTA

20. Find the average value of $f(x)=\frac{x-2}{x-1}$ from $x=2$ to $x=5$.

A) $\frac{1}{4}$ B) $1-\ln\sqrt[3]{4}$ C) $3-\ln 4$ D) $3-\ln\frac{5}{2}$ E) NOTA

21. Let $g(x)=[f(x)]^3$. If $g(7)=64$ and $f'(7)=12$, what is $g'(7)$?

A) 48 B) 144 C) 192 D) 576 E) NOTA

22. Evaluate: $\ln 16 \int 2^x \, dx$

A) 2^x+C B) $2^{x+1}+C$ C) $2^{x+2}+C$ D) $2^{x+3}+C$ E) NOTA

23. Find the slope of the line tangent to $y = \log_4 x$ at x = 2.

A) $-\ln 16$ B) $\ln \frac{e}{16}$ C) $\frac{1}{\ln 16}$ D) $\frac{1}{2}$ E) NOTA

24. Let u and v be functions of a variable x. Evaluate the derivative of u^v.

A) $u^v(1+\ln u)$ B) $vu^{v-1}\frac{du}{dx}+u^v \ln(u)\frac{dv}{dx}$

C) $vu^{v-1}\frac{du}{dx}$ D) $vu^{v-1}\frac{dv}{dx}+u^v \ln(u)\frac{du}{dx}$ E) NOTA

25. Let $h(x)=\frac{f(x)}{g(x)}$. If $f(a)=2$, $f'(a)=-1$, $f''(a)=2$, $g(a)=1$, $g'(a)=2$, and $g''(a)=5$, evaluate $h''(a)$.

A) $\frac{2}{5}$ B) 2 C) 12 D) 18 E) NOTA

26. The particle's position is given by the function $s(t)=t^4+t\cos t$. Find the function's jerk (in units per second cubed) at $t=\pi$. Note: Jerk is defined as the third derivative of the position function.

A) $24-\pi$ B) $24\pi+3$ C) $12\pi^2+\pi$ D) $4\pi^3-1$ E) NOTA

27. To which of the following does Rolle's Theorem apply?

A) $f(x)=|x|$ on the interval $[-3,3]$ B) $f(x)=\frac{2}{x^2}$ on the interval $[-2,2]$

C) $f(x)=|\cos x|$ on the interval $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

D) $f(x)=x^3+5x^2-x+2$ on the interval $[-2,1.5]$ E) NOTA

28. Evaluate: $\lim_{x\to\infty} a\left(1+\frac{b}{x}\right)^{cx}$

A) e^{-bc} B) ae^{-bc} C) e^{bc} D) ae^{bc} E) NOTA

29. Find $\frac{d^2}{dx^2}(x\cos x\sin x)$

A) $2\cos(4x)$ B) $2(\cos(2x)-\sin(2x))$
C) $2x(\cos(2x)-\sin(2x))$ D) $2(\cos(2x)-x\sin(2x))$ E) NOTA

30. Evaluate: $\int\frac{9}{36+x^2}\,dx$

A) $\arctan\frac{x}{6}+C$ B) $\frac{1}{6}\arctan\frac{x}{6}+C$
C) $\frac{3}{2}\arctan\frac{x}{6}+C$ D) $\frac{1}{4}\arctan\frac{x}{3}+C$ E) NOTA