Calculus Individual

Palm Harbor University High School
March 18th 2000

The abbreviation "**NOTA**" denotes "**None Of These Answers**"

1. Let $f(x)=\begin{cases}(2x-2)^3, & x<2\\ g(x), & x>2\end{cases}$ and

   I. $g(x)=x^3$
   II. $g(x)=2x$
   III. $g(x)=\dfrac{x^2+4x-12}{x-2}$

   For which of the functions $g(x)$ does $\lim\limits_{x\to 2} f(x)$ exist?

   a) I, III b) I only c) I, II, III d) III only e) NOTA

2. Evaluate the integral $\int \sin^3 x\,dx$

   a) $\dfrac{\sin^4 x}{4}+C$ b) $\dfrac{\cos^3 x}{3}-\cos x+C$
   c) $\dfrac{\cos^4 x}{4}+C$ d) $\dfrac{\sin^4 x}{4}-\sin x+C$ e) NOTA

3. What is the equation for the normal line to $f(x)=x^2$ at $x=3$?

   a) $y-6=9(x-3)$ b) $y-9=6(x-3)$
   c) $y-6=\frac{-1}{9}(x-3)$ d) $y-9=\frac{-1}{6}(x-3)$ e) NOTA

4. Find the first derivative of $P(x)=\dfrac{x^2-1}{3-2x}$

   a) $\dfrac{-5x^2+6x+1}{(3-2x)^2}$ b) $\dfrac{-2x^2+6x-2}{(3-2x)^2}$
   c) $\dfrac{6x^2-6x+2}{(3-2x)^2}$ d) $\dfrac{6x^2-2x+2}{(3-2x)^2}$ e) NOTA

5. Use the Mean Value Theorem for Integrals to find the value(s) of c for this function

$$f(x) = \begin{cases} \dfrac{(x-4)^2}{2}, & x \le 2 \\ \dfrac{4x^2 - 14x + 12}{x - 2}, & x > 2 \end{cases} \quad \text{on the interval } [0, 4]$$

a) 1 b) $4 + 2\sqrt{3}$

c) $4 - 2\sqrt{3}, \frac{17}{6}$ d) $4 - \frac{4\sqrt{6}}{3}, \frac{17}{6}$ e) NOTA

6. Determine the interval of convergence of the series $\sum_{k=1}^{\infty} \frac{2^k}{\sqrt{k}}(x+2)^k$

a) $[\frac{-5}{2}, \frac{-3}{2}]$ b) $(\frac{-3}{2}, \frac{-1}{2}]$ c) $[\frac{-5}{2}, \frac{-3}{2})$ d) $(\frac{-1}{2}, \frac{1}{2}]$ e) NOTA

7. Newton's Law of Cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the constant temperature of the surrounding medium, called the *ambient temperature*. A steaming cup of hot chocolate with marshmallows obeying Newton's Law is served to your friend, Big Pun. The 185°F beverage is placed on a table in a room with a constant temperature of 65°F. Two minutes later, the temperature of the hot chocolate has dropped to 155°F. If Big Pun can't drink the hot chocolate until it has cooled to 105°F, how much longer must he wait to consume the hot chocolate?

a) $2\ln 4$ b) $\dfrac{\ln\frac{1}{3}}{2\ln\frac{3}{4}}$ c) $\dfrac{\ln\frac{1}{9}}{\ln 3 - \ln 4}$ d) 4 e) NOTA

8. Determine the average value of $y = 4x^3 + 4$ on the interval $3 \ge x \ge 0$

a) -31 b) 31 c) $\frac{93}{2}$ d) 93 e) NOTA

9. Evaluate the definite Integral $\int_2^3 \frac{1}{(x+1)(x-1)}\,dx$

a) $\ln\frac{2}{3} - 1$ b) $\ln\frac{3}{2}$ c) $\frac{1}{2}\ln\frac{2}{3}$ d) $\frac{1}{2}\ln\frac{3}{2}$ e) NOTA

10. Find the difference in volume of the solids of revolution formed when the region enclosed by the x-axis, y-axis, and $f(x)=2-2x$ is rotated about both the x and y-axes.

a) $\frac{2}{3}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{4\pi}{3}$ e) NOTA

11. $\lim_{y\to\infty} \frac{1}{y}\int_0^y \sin\left(\frac{1}{t+1}\right)dt$

a) 0 b) sin 1 c) 1 d) ∞ e) NOTA

12. On what interval is $\frac{x^2}{x^2-9}$ increasing and concave down ?

a) $(-3, 3)$ b) $(-3, 0)$

c) $(-3, 0)\cup(3,\infty)$ d) $(-\infty, -3)\cup(3,\infty)$ e) NOTA

13. Find the solution of $y' - \frac{2}{x}y = x$ that satisfies the initial condition $y(1)=1$

a) $y=x+1,\ x\neq 1$ b) $y=\ln x+1,\ x\neq 0$

c) $y=x^2\ln x+1,\ x\neq 1$ d) $y=x^2\ln x+x^2,\ x\neq 0$ e) NOTA

14. A small object called a POD is dropped from a building and hits the ground 6 seconds later. If gravity is 32 feet per second per second, from what height was the POD dropped?

a) 176.4 ft. b) 192 ft. c) 576 ft. d) 1152 ft. e) NOTA

15. What is the total area between the x-axis and the equation for the linearization of $y=x^2e^x$ at $x=1$ from $0\leq x\leq 2$?

a) $2e$ b) $\frac{10}{3}e$ c) $4e$ d) $2e^2+2$ e) NOTA

16. The base of a solid is the region between the parabolas $x=y^2$ and $x=3-2y^2$. Find the volume of the solid given that the cross sections perpendicular to the x-axis are squares.

a) 6 b) 4 c) 2 d) 1 e) NOTA

17. Find the area bounded by $y=\dfrac{10}{1+x^2}$ and the x-axis on the interval $[1,\infty)$

a) $\dfrac{5\pi}{2}$ b) $5\pi-\dfrac{10\pi}{3}$

c) 5π d) ∞ e) NOTA

18. Sammy J, located at (0, 0), is standing in front of a beach house watching Dalhoff, located at (−528, 0), windsurf by on a *1999 Bic Vivace Carbon Board* with a 7.5 m^2 sail at 4.5 miles per hour on a due north heading. If D-hoff continues north, find the rate at which the distance between Sammy J and D-hoff is increasing after one minute.

a) 2.7 $\frac{ft.}{sec}$ b) 3.5 $\frac{ft.}{sec}$ c) 237.6 $\frac{ft.}{sec}$ d) 316.8 $\frac{ft.}{sec}$ e) NOTA

19. Find the surface area of revolution about the x-axis of $y=\dfrac{10}{1+x^2}$ on the interval $[1,\infty)$

a) $2\pi\int_1^\infty x\sqrt{1+(y')^2}\,dx$ b) $2\pi\int_1^\infty x\sqrt{1+x^2}\,dx$

c) $\pi\int_1^\infty \frac{10}{1+x^2}\sqrt{1+(y')^2}\,dy$ d) $2\pi\int_1^\infty \frac{10}{1+x^2}\sqrt{1+(y')^2}\,dx$ e) NOTA

20. What are the coordinates of the centroid of the area enclosed by the parametric equations

$x(\theta)=\frac{4}{5}\cos\theta+\frac{5}{2}$ $y(\theta)=\frac{15}{8}\sin\theta-\frac{7}{4}$

a) $\left(\frac{4}{5},\frac{15}{8}\right)$ b) $\left(\frac{15}{8},\frac{4}{5}\right)$ c) $\left(\frac{5}{2},\frac{-7}{4}\right)$ d) $\left(\frac{-7}{4},\frac{5}{2}\right)$ e) NOTA

21. Let P be the region enclosed by the graphs of $f(x)=x^2-3$ and $f(x)=x^3-x-2$. The line $x=b$ separates the region into two parts of equal area. Find the value of b to the nearest ten-thousandth.

a) -0.4351 b) -0.2285 c) 0.3142 d) 0.5123 e) NOTA

22. Use differentials to estimate the value of $y=x^{\frac{2}{3}}$ when $x=26$

a) 7 b) $\frac{79}{9}$ c) $\frac{83}{9}$ d) 11 e) NOTA

23. Find D_x^2 of $(x+y)(x-y)=-(1-xy)(-x-y)$

a) $\dfrac{2-x-y}{(1-x)^2}$ b) $\dfrac{2(1+y)}{(1-x)^2}$

c) $\dfrac{(1+y)}{(x-1)}$ d) $\dfrac{(1-x+y)}{(x-1)^2}$ e) NOTA

24. The radius of curvature is the reciprocal of the curvature of a function at a point. If curvature, $\boldsymbol{K}$, is given by the formula $\boldsymbol{K}=\dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$, find the radius of curvature of the space curve $\mathbf{r}(t)=t\,\mathbf{i}+\frac{1}{2}t^2\,\mathbf{j}$ at $t=\sqrt{3}$

a) $\frac{1}{2}$ b) 2 c) 8 d) $6\sqrt{3}+10$ e) NOTA

25. Evaluate $\lim\limits_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{3i}{n}\right)^2\left(\frac{3}{n}\right)$

a) 39 b) 41 c) 43 d) ∞ e) NOTA

26. A new windsurfer board decreases in value at a rate that is given by the function $w(t)=1000e^{-t}$ here t is in years on the interval $[0,\infty)$. If $w(t)$ is in dollars, what is the total decrease in value of the board **ln 2** years after it is bought?

a) 500 b) $500\ln 2$ c) 1000 d) $1000\ln 2$ e) NOTA

27. $\lim_{x \to \infty} f(x) = \sqrt{(x^2 + 4x)} - x$

a) 0 b) 1 c) $\sqrt{2}$ d) ∞ e) NOTA

28. Evaluate the Integral $2\int_0^{2\sqrt{2}} \int_0^x \int_0^y \mathrm{dz\, dy\, dx}$

a) 3

b) $\int_{-1}^{1} (x+1)^2 \mathrm{dx}$

c) $\frac{7}{3}$

d) $\frac{26}{3}$

e) NOTA

29. Find $\frac{\partial f}{\partial z}(x, y, z)$ of $f(x, y, z) = xyz^2$

a) $2xz$ b) $2xyz$ c) yz^2 d) xy e) NOTA

30. Estimate the following integral using the Trapezoidal Rule $\int_0^{\frac{\pi}{2}} \sin x \, \mathrm{dx}$ with $n = 1$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π e) NOTA