Palm Harbor Invitational – February 2005 Algebra II Team Question # 1

What is the area of the figure made by connecting the centers of the following four figures?

A: $3x^2 - 12x + 24y^2 + 384y = -1476$

B: $5x^2 - 20x - 6y^2 - 48y = 106$

C: $x^2 - 8x + y^2 + 8y = 31$

D: The square with vertices at (6, -6), (2, -6), (2, -10), and (6, -10).

Palm Harbor Invitational – February 2005 Algebra II Team Question # 2

There are two parallel piers jutting out into the Gulf of Mexico 100 m apart in an area where the coastline is perfectly straight. One is 75 m long, and the other is 50 m long. Ian and Eileen decide to race from the tip of the 75 m pier to the tip of the 50 m pier, however, Ian will swim between the two without returning to shore, and Eileen will swim to the shore and then back out to the tip of the other pier. If they both choose their shortest possible path, and if Eileen can swim at a speed of 5 m/s, while Ian can only swim at a speed of 4 m/s, by how many seconds will Ian beat Eileen in the race? (Round your answer to four decimal places.)

Palm Harbor Invitational – February 2005 Algebra II Team Question # 3

Given:

$$2\ln A = 0$$
$$B = (\log_D(D+2))(\log_{D+2} D) + 5$$
$$C = \ln(\ln e^{\ln e}) + \ln e^e$$
$$\log_3(D-2) + \log_3(D+3) = 0$$

Find: $\dfrac{A+B+2D}{C}$ (Exact Answer)

Palm Harbor Invitational – February 2005 Algebra II Team Question # 4

Given:

$$f(x) = 3 \times 2^x$$
$$g(x) = x^2 - 8$$
$$h(x) = 3x + 20$$
$$s(x) = \ln(x+4)$$

Find:

$$f(f^{-1}(x)) + g(g^{-1}(x)) + h(h^{-1}(x)) + s(s^{-1}(x))$$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 5

Quang loves pez. He has a giant snoopy pez dispenser in his room. That large pez dispenser dispenses packets of pez instead of just individual pieces of pez candy. Each packet of pez has 10 pieces of pez candy. The holder itself holds 15 pez packets. If for any given piece of pez candy in any given packet of pez the probability of that piece of pez being orange is 1/6. What is the expected number of orange pieces of pez candy found in the large snoopy pez dispenser?

Palm Harbor Invitational – February 2005 Algebra II Team Question # 6

Find the value of the units digit of the following expression:

$$999^{111} + 888^{222} + 777^{333} + 666^{444} + 555^{555} + 444^{666} + 333^{777} + 222^{888} + 111^{999}$$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 7

Determine the equation of the line which contains the point (7, -2), and is perpendicular to the line that contains the points (1, 6) and (-3, 8). Answer in the form $ax + by = c$, where *a, b,* and *c* are relatively prime integers and *a* is positive.

Palm Harbor Invitational – February 2005 Algebra II Team Question # 8

Given: $$\prod_{x=a}^{b} f(x) = f(a) \times f(a+1) \times (a+2) \times \ldots \times f(b-1) \times f(b)$$

$$\text{A} = \sum_{y=7}^{25} (2y + 14) \qquad \text{B} = \sum_{j=3}^{13} 3(2)^j$$

$$\text{C} = \sum_{k=4}^{\infty} 3\left(\frac{1}{2}\right)^k \qquad \text{D} = \prod_{x=1}^{35} i^x \quad (\text{ Where } i = \sqrt{-1}\ )$$

Find:

$$A + B + C + D$$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 9

Consider Pascal's Triangle in which the topmost row (the row containing a single one) is called row 0, followed by the next row (which contains two 1's) labeled row 1, and so forth. What would be the sum of the elements in the 12^{th} row of Pascal's Triangle if each element were to be increased by 4 and then multiplied by 2?

Palm Harbor Invitational – February 2005 Algebra II Team Question # 10

Given the equation: $3x^2 - 7x + 28 = 1$

A = the solution to the equation that resides in quadrant I of the complex number plane
B = the solution to the equation that resides in quadrant II of the complex number plane
C = the discriminant

Find: $\dfrac{A+B}{C}$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 11

Given:

$$A = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{...}}}}$$

$$B = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{...}}}$$

Find: $A + 2B$ (Exact Answer)

Palm Harbor Invitational – February 2005 Algebra II Team Question # 12

Given:

$$\begin{vmatrix} A & 4 \\ 2 & A \end{vmatrix} = 8 \quad , \; A > 0$$

$$\begin{bmatrix} 2 & m \\ -8 & k \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2k & m \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & B \end{bmatrix}$$

Find: $A + B$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 13

Find: $\displaystyle\sum_{i=1}^{127} \log_2 \frac{i}{i+1}$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 14

Given: $\dfrac{1}{x^2+2x-8} = \dfrac{A}{x+4} + \dfrac{B}{x-2}$

Find: $A + B$

Palm Harbor Invitational – February 2005 Algebra II Team Question # 15

Find the 6^{th} term of the binomial expansion of $\left(2x^3 + 3y^6\right)^{11}$.