Algebra II Team Solutions

Question 1

$$\frac{20\text{mi} + 20\text{mi}}{20\text{min} + 50\text{min} + 60\text{min}} = \frac{4}{13}\text{ Miles per minute}$$

Question 2

$$\log_b c^x + d\log_b a = b\log_b a$$

$$x\log_b c = b\log_b a - d\log_b a$$

$$x = \frac{(b-d)\log_b a}{\log_b c}$$

$$x = (b-d)\log_c a$$

$$x = 4(25)$$

$$x = 100$$

Question 6

A

$$x^2 - 10x + y^2 - 24y = 0$$

$$(x-5)^2 + (y-12)^2 = 25 + 144$$

$$(x-5)^2 + (y-12)^2 = 13^2$$

$$A = \pi r^2$$

$$= \pi(13^2)$$

$$= 169\pi$$

B

This is a triangle with x-intercept 2 and y intercept 6.

$$Area = \frac{1}{2}(2)(6)$$

$$Area = 6$$

B=6

C

$$C = \pi ab$$

$$= \pi(\sqrt{5})(\sqrt{125})$$

$$= 25\pi$$

$$\frac{169\pi + 6}{25\pi}$$

Question 3

A

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

A = -1

$$B = 52 + 53 + 65 + 32$$

B = 202

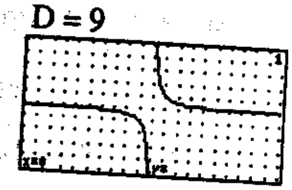
$$C = 60 - 4(15)$$

C = 0

D

hyperbola

D = 9



A+B+C+D

-1+202+0+9=210

Question 4

$$\begin{bmatrix} 2 & 8 & 5 \\ 3 & 6 & 4 \\ 8 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 4 \\ 5 & 3 & 3 \\ 7 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 77 & 56 & 52 \\ 61 & 52 & 46 \\ 27 & 59 & 43 \end{bmatrix}$$

Hence the answer is 52.

Question 5

If $f^{-1}(a) = 2$, then a is $f(2)$

$$f(2) = \left(\left(1 + \sqrt{2(2)}\right) + \frac{1}{2}\right)^2 + 3^2$$

$$f(2) = 3.5^2 + 3^2$$

$$f(2) = 21.25$$

$$a = 21.25$$

Question 7

The coefficient of the seventeenth term of $\left(\log_{20736}\left(12^{a}\right)+x\right)^{21}$

$$\left(\log_{20736}\left(12^{a}\right)+x\right)^{21}=\left(a\left(\log_{20736}12\right)+x\right)^{21}$$
$$=\left(\frac{1}{4}a+x\right)^{21}$$

$${}_{21}C_{16}\left(\left(\frac{1}{4}a\right)^{5}\times x^{16}\right)=\frac{20349}{1024}a^{5}x^{16}$$

Hence, $\frac{20349}{1024}$.

Question 8

The sum of the foci of an ellipse to any point on the ellipse is always the same.

$$0=25x^{2}+36y^{2}-200x-216y-176$$
$$0=25\left(x^{2}-8x+16\right)+36\left(y^{2}-6y+9\right)-176-400-324$$
$$900=25(x-4)^{2}+36(y-3)^{2}$$
$$1=\frac{(x-4)^{2}}{6^{2}}+\frac{(y-3)^{2}}{5^{2}}$$

The point (4,8) is in the ellipse:

$$1=\frac{(x-4)^{2}}{6^{2}}+\frac{(y-3)^{2}}{5^{2}}$$
$$1=\frac{(4-4)^{2}}{6^{2}}+\frac{(8-3)^{2}}{5^{2}}$$
$$1=\frac{0}{36}+\frac{25}{25}$$
$$1=1$$

The sum of foci is the same as the length of the major axis, which is 12.

Question 9

20+10+5+2.5+1.25...

$$\frac{a_{n}}{1-r}=\frac{20}{1-0.5}=\frac{20}{0.5}=40$$

Question 10

You are given 3 points of a parabola: (0,60)(3,0)(1,18). From these, 3 simultaneous equations can be derived:

$$60=c$$
$$0=9a+3b+c$$
$$18=a+b+c$$

Solving, the parabola is:

$$y=11x^{2}-53x+60$$

Question 11

1+1, 1+2...1+10 = 10 combinations for a 1 on the 20 sided die.
Hence, 10x20 combinations total.
The numbers 2 to 30 are possible sums.
2 has only one possibility (1+1) as does 30 (20+10).
3 has 2 possibilities (1+2 and 2+1) as does 29 (20+9 and 19+10).
We see this pattern continues ending at the number 11 (10 possibilities: 1+10, 10+1, 2+9, 9+2, 3+8, 8+3, 4+7, 7+4, 5+6 and 6+5) and with 21 (10 possibilities: 20+1, 19+2, 18+3, 17+4, 16+5, 15+6, 14+7, 13+8, 12+9, and 11+10).
For the numbers between 11 and 21 (exclusive), there is a different pattern. For each of those numbers, here are the combinations:

12: 11+1, 10+2, 2+10, 9+3, 3+9, 8+4, 4+8, 7+5, 5+7, 6+6.
13: 12+1, 11+2, 10+3, 3+10, 9+4, 4+9, 8+5, 5+8, 7+6, 6+7.
14: 13+1, 12+2, 11+3, 10+4, 4+10, 9+5, 5+9, 8+6, 6+8, 7+7.
15: 14+1, 13+2, 12+3, 11+4, 10+5, 5+10, 9+6, 6+9, 8+7, 7+8.
16: 15+1, 14+2, 13+3, 12+4, 11+5, 10+6, 6+10, 9+7, 7+9, 8+8.
17: 16+1, 15+2, 14+3, 13+4, 12+5, 11+6, 10+7, 7+10, 9+8, 8+9.
18: 17+1, 16+2, 15+3, 14+4, 13+5, 12+6, 11+7, 10+8, 8+10, 9+9.
19: 18+1, 17+2, 16+3, 15+4, 14+5, 13+6, 12+7, 11+8, 10+9, 9+10.
20: 19+1, 18+2, 17+3, 16+4, 15+5, 14+6, 13+7, 12+8, 11+9, 10+10.

There are 10 possibilities for the numbers 11 through 21. Hence, the answer is 11 through 21.

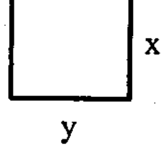
Question 12

$$\text{Area} = xy$$
$$400 = 2x + 2y$$

$$y = 200 - x$$
$$\text{Area} = x(200 - x)$$
$$\text{Area} = 200x - x^2$$

$$\frac{-b}{2a} = \frac{-200}{-1(2)}$$
$$= 100$$

$$y = 200 - 100$$
$$y = 100$$



$$Area = 100 \times 100$$
$$Area = 10000 \text{ ft}^2$$

Question 13

$$\log_{65}|3+4i| + \log_{65}|5+12i|$$
$$\log_{65}(|3+4i| \times |5+12i|)$$
$$\log_{65}(5 \times 13)$$
$$\log_{65}(65)$$
$$1$$

Question 14

Remember to sort the polynomial by order of the degree.

$$
\begin{array}{r|l}
 & x^2 - 2x + 12 \\
x+4 & x^3 + 2x^2 + 4x + 3 \\
 & \underline{x^3 + 4x^2} \\
 & -2x^2 + 4x \\
 & \underline{-2x^2 - 8x} \\
 & 12x + 3 \\
 & \underline{12x + 48} \\
 & -45
\end{array}
$$

From this, the answer is:

$$x^2 - 2x + 12 - \frac{45}{x+4}$$

Question 15

$$(6x+1)\left[\left(\frac{3}{2}\right)\times\left(4x+\frac{2}{3}\right)\right]^4=(6x+1)\times\left(\frac{3}{2}\right)^4\times\left(4x+\frac{2}{3}\right)^4$$

$$=(6x+1)\times\left(\frac{3}{2}\right)^4\times\left(\frac{2}{3}\right)^4\times(6x+1)^4$$

$$=(6x+1)^5$$

The fourth term would be ${}_5C_1(6x)^4(1)^1$.

Hence the coefficient is $5\times 6^4=6480$.

Algebra II Team Answers

1. $\frac{4}{13}$ Miles per minute
2. $x=100$
3. 210
4. 52
5. $a=21.25$
6. $\frac{169\pi+6}{25\pi}$
7. $\frac{20349}{1024}$
8. 12
9. 40
10. $y=11x^2-53x+60$
11. The numbers 11 through 21.
12. $Area=10000\text{ ft}^2$
13. 1
14. $x^2-2x+12-\frac{45}{x+4}$
15. 6480