

2004 Palm Harbor Invitational Algebra II Individual

Directions: Mark the best answer on your scantron. Choice E, NOTA, stands for None Of These Answers. When appropriate, $i = \sqrt{-1}$.

1. Find $a + b - c$ where a , b , and c are roots of the following polynomial in order from greatest to least.

$$y = x^3 + 4x^2 - 7x - 10$$

- A) -8 B) 2 C) 6 D) 8 E) NOTA

2. Find the sum of the coefficients of the terms (including the constant terms) in the expansion of $(3 - 2x)^3$.

- A) -26 B) -8 C) 1 D) 19 E) NOTA

3. Which of the following statements are true?

1) The function $f(x) = x^3$ is odd.

2) The number of possible negative real roots of $f(x) = 5x^5 + kx^4 - 2x^3 + 9x^2 - 18$ is 3, where $k > 0$.

3) The graph of $4x^2 - 3y^2 - 5x + 6 = 0$ is a hyperbola.

4) The arithmetic mean of 2 and 3 is 2.5.

- A) 1,3,4 only B) 2,3 only C) 1,2 only D) 3,4 only E) NOTA

4. Mr. Rob leaves a cup of coffee in his R&D cubicle and it begins to cool at an exponential rate. Three minutes after he puts it down on his desk, the temperature of the coffee is at 110°F . Eight minutes after he put it down, the coffee is at 95°F . What was the temperature 2 minutes after Mr. Rob first set down the coffee cup? (to the nearest hundredth)

- A) 108.02°F B) 113.27°F C) 118.36°F D) 118.37°F E) NOTA

5. Find $\frac{y}{x}$ to the nearest tenth.

$$\begin{bmatrix} 2 & 0 & 5 \\ x & -3y & -4 \end{bmatrix} \begin{bmatrix} 3x & -2 \\ -6 & -y \\ 9 & x \end{bmatrix} = \begin{bmatrix} 63 & 11 \\ 27x & 57 \end{bmatrix}$$

- A) 0.6 B) 1.3 C) 1.7 D) 2.2 E) NOTA

6. Find the equation for the axis of symmetry for $f(x) = 3x^2 - 7x + 12$.

- A) $x = -\frac{7}{6}$ B) $y = -\frac{7}{6}$ C) $x = \frac{7}{6}$ D) $y = \frac{7}{6}$ E) NOTA

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7. Solve for x : $\log(5^x) - \log(5^{x+4}) + \log(5^3) = \log(5^{x-3})$

- A) 0 B) 2 C) 4 D) 6 E) NOTA

8. Three traffic lights on U.S. 19, all of which operate independently of each other, impede Luis's drive to school. The probability of him being stopped at the first light is 0.5, at the second light is 0.3, and at the third light is 0.8. Luis will be late for school if he gets stopped at all three lights. What is the probability of him getting stopped by exactly two traffic lights?

- A) 0.07 B) 0.43 C) 0.59 D) 0.79 E) NOTA

9. Given a , b , and c are the next three terms in the sequence, respectively, find $b - (c - a)$.
-1, 0, 3, 8, 15, 24, 35, 48 ...

- A) -82 B) -13 C) 44 D) 116 E) NOTA

10. Adrienne drops her tennis ball from a height of 8 feet which starts to bounce 60% of its previous height each time. What is the total vertical distance in inches that the ball will travel?

- A) 16 B) 32 C) 288 D) 384 E) NOTA

11. Find the 4th term of the binomial expansion of $(3x + 2y)^6$ if listed in decreasing powers of x .

- A) $4860x^4y^2$ B) $2160x^4y^2$ C) $4320x^3y^3$ D) $3240x^3y^3$ E) NOTA

12. Find the equation of the line in the xy -plane which is equidistant from the points $(6, 8)$ and $(-8, -2)$.

- A) $7x + 5y = 8$ B) $5x + 7y = 16$
C) $7x + 5y = 74$ D) $5x - 7y = 0$ E) NOTA

13. If $f^{-1}(x)$ is the inverse of $f(x)$ and $f(x) = x^3 + 2$, then what is the product of $f(x + 2)$ and $f^{-1}(x)$ at $x = 10$?

- A) 2746 B) 3027 C) 3460 D) 4098 E) NOTA

14. What is the remainder of $\frac{6x^3 + 5x^2 - 4x + 8}{2x + 3}$?

- A) -7 B) 0 C) 3 D) 5 E) NOTA

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15. Simplify: $\sum_{j=1}^{10} (3j+2) + \sum_{j=0}^{10} (3 \times 2^j)$

- A) 185 B) 3,069 C) 3,254 D) 6,326 E) NOTA

16. Given y is jointly proportional to x and $\log z$, and inversely proportional to the square root of q , what is the value of the constant of proportionality when $y=12$, $x=50$, $q=16$, and $z=100$?

- A) 0 B) 0.48 C) 0.96 D) 1 E) NOTA

17. Find the vertical asymptote(s) of the following function.

$$f(x) = \frac{2x^2 - x - 15}{3x^2 - 5x - 12}$$

- A) $x = 3, x = \frac{4}{3}$ B) $x = 3$
 C) $x = -3, x = -\frac{4}{3}$ D) $x = -\frac{4}{3}$ E) NOTA

18. Evaluate the determinant of the following matrix: $\begin{bmatrix} 5 & -3 & 8 \\ 2 & 6 & 9 \\ -7 & 4 & 4 \end{bmatrix}$

- A) -342 B) 0 C) 445 D) 553 E) NOTA

19. Given $\log 32 - \log 4 + \log 1024 = \log (64x)$, find $x + \log x$ to the nearest hundredth.

- A) 68.37 B) 130.11 C) 143.29 D) 256.22 E) NOTA

20. Find the value of $A - B$: $\frac{5x+3}{x^2+6x-16} = \frac{A}{x-2} + \frac{B}{x+8}$

- A) -2.4 B) -0.4 C) 1.6 D) 3.5 E) NOTA

21. Which of the following is a cubic function whose roots include 6 and $4 + i\sqrt{14}$, where $i = \sqrt{-1}$.

- A) $f(x) = x^3 - 14x^2 + 78x - 180$ B) $f(x) = x^3 - 14x^2 + 23x - 30$
 C) $f(x) = x^3 - 4x^2 + 8x - 120$ D) $f(x) = 3x^3 - 14x^2 + 36x - 36$ E) NOTA

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22. Given: $|x + yi| = 15$ and $|(x + 1) + 2yi| = 26$ where $x > 0$, $y > 0$, and $i = \sqrt{-1}$. Find xy .

- A) 75 B) 96 C) 108 D) 390 E) NOTA

23. How many distinct permutations can be formed from the word, DICTIONARY?

- A) 181,440 B) 604,800 C) 1,814,400 D) 3,628,800 E) NOTA

24. Farmer Warren needs to build a rectangular fence to keep in his cattle. He has enough wood and wire to build 240 feet of fence. The fence is going to be built against a wall which will be the fourth side of the fence and is long enough to be the side for any fence that he can make. What is the maximum possible area of the fenced in enclosure in square feet?

- A) 6,400 B) 7,200 C) 8,400 D) 9,600 E) NOTA

25. Find the discriminant of the polynomial that is the result of $\frac{5x^3 - 29x^2 + 52x - 30}{x - 3}$.

- A) 2 B) 4 C) 9 D) 16 E) NOTA

26. What is the units digit of $526^{857} + 2158^{3069} - 443^{256} + 852^{567}$?

- A) 1 B) 2 C) 3 D) 4 E) NOTA

27. Find the minimum value of $f(x) = \begin{cases} |2x - 4|, & x \neq 2 \\ 3, & x = 2 \end{cases}$

- A) -4 B) -3.9999 C) 0 D) 3 E) NOTA

28. How many 0's are at the end of $128!$?

- A) 31 B) 32 C) 33 D) 34 E) NOTA

29. If $f(6) = 32$, $f(8) = 40$ and $f(x)$ is a linear function, find $f(2)$.

- A) 4 B) 8 C) 16 D) 24 E) NOTA

30. Evaluate: $\sum_{x=14}^{62} (3x - 5)$

- A) 5,341 B) 5,375 C) 5,488 D) 5,525 E) NOTA