Palm Harbor High School Invitational Test 2000

Algebra II Individual Answers

1. D
2. B
3. C
4. B
5. D
6. B
7. C
8. C
9. B
10. B
11. B
12. A
13. C
14. D
15. C
16. E
17. B
18. A
19. A
20. D
21. C
22. B
23. A
24. C
25. A
26. E
27. D
28. C
29. E
30. D

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Algebra II Individual Solutions

1. $f(3) = 3^7 + 9\cdot 3^6 - 5\cdot 3^4 + 2\cdot 3^2 - 9\cdot 3 + 3 = 8337$ **D**

2. 7^x has period 4 — 9^x has period 2 — 1^x has period 1
   7^1 ends in 7 — 9^1 ends in 9 — 1^x ends in 1
   7^2 ends in 9 — 9^2 ends in 1
   7^3 ends in 3 — 9^{2000} ends in 1
   7^4 ends in 1
   7^{1998} ends in 9 — $9 + 1 + 1 = 11$ ends in 1 **B**

3. $f(-5) = x^7 + 5x^6 - 5x^5 + 4x^4 + 3x^3 - 9x^2 + 13x + 36 =$
   $(-5)^7 + 5(-5)^6 - 5(-5)^5 + 4(-5)^4 + 3(-5)^3 - 9(-5) + 13(-5) + 36 = 17496$ **C**

4. Since letters occur twice $\frac{11!}{2!2!} = 9979200$ **B**

5. $3x^3 + 4x^2 - 6x - 7 = 0 \quad \frac{-c}{d} = \frac{6}{7}$ **D**

6. $\sum_{x=1}^{120} x^2 = \frac{n(n+1)(2n+1)}{6} = \frac{120(121)(241)}{6} = 583220$
   $\sum_{x=1}^{27} x^2 = \frac{27(28)(55)}{6} = 6930 \quad 583220 - 6930 = 576290$ **B**

7. $\frac{2}{3}x = \frac{3}{4}y + \frac{5}{6} \qquad 8x = 9y + 10 \qquad 8x - 9y - 10 = 0 \quad 8 \cdot 9 \cdot 10 = -11$ **C**

8. $\frac{x^{2.5}}{\sqrt{x}} = x^{2.5} - x^{.5} = x^2$ **C**

9. $x^4 + 3x^2 - 4$ Since any real factors will be from ± 1, ± 2, or ± 4
   Try 1 - $1^4 + 3\cdot 1^2 - 4 = 1 + 3 - 4 = 0$ so 1 is a root
   Try -1 - $(-1)^4 + 3\cdot(-1)^2 - 4 = 1 + 3 - 4 = 0$ so -1 is a root
   1] 1 0 3 0 -4 — -1] 1 1 4 4
   1 1 4 4 — -1 0 -4
   1 1 4 4 0 — $x^3 + x^2 + 4x + 4 = 0$ — 1 0 4 0 — $x^2 + 4 = 0$
   $\frac{0 \pm \sqrt{0-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$ 2i and -2i are roots.
   $\therefore x^4 + 3x^2 - 4 = (x-1)(x+1)(x-2i)(x+2i)$ **B**

10. $-23 + y = 2 \quad y = 25 \quad g(2) = \log_5 25 \quad g(2) = 2 \quad f(x) = 2 \quad \log_x 5 = 2 \quad x^2 = 5 \quad x = \pm\sqrt{5}$
    But $-\sqrt{5}$ doesn't work. $\sqrt{5}$ **B**

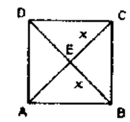
11. $3x + 3yi = 9 + 6i \qquad 3x = 9 \quad x = 3 \qquad 3yi = 6 \quad y = 2 \qquad 3 + 2 = 5$ **B**

12. $\frac{5!}{2} = \frac{120}{2} = 60$ **A**

13. I. true
    II. false
    III. true **C**

14. $x = 1 \quad 17 - 8 - 3 + 12 + 4 - 2 + 3 - 1 = 22$
    $x = 2 \quad 7925$ — $x \to \infty, f(x) \to \infty$
    $x = -1 \quad -17 + 8 - 3 - 12 + 4 + 2 - 3 - 1 = -22$ — $x \to -\infty, f(x) \to -\infty$ **D**

15. $x^2 + x^2 = 5^2 \qquad 2x^2 = 25 \qquad x^2 = \frac{25}{2} \qquad x = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ **C**



16. $1\cdot 5\cdot 9 + 2\cdot 6\cdot 7 + 3\cdot 4\cdot 8 - 7\cdot 5\cdot 3 - 8\cdot 6\cdot 1 - 9\cdot 4\cdot 2 = 45 + 84 + 96 - 105 - 48 - 72 = 0$ Inverse is undefined **E**

17. 16 notes $16^5 = 1{,}048{,}576$ **D**

18. $\sum_{2}^{700} x = \sum_{x=-700}^{-2} x \quad \sqrt[3]{0} = 0$ **A**

19. $\sqrt{290} + \frac{10+29}{2} + \frac{2}{\frac{1}{10} + \frac{1}{29}} \approx 51.40$ rounding down, 51. **A**

20. $\frac{4+8}{104} = \frac{12}{104} = \frac{3}{26}$ **D**

21. I. true
    II. false; $\log_y x^z = z \bullet \log_y x$; $(\log_y x)^z \neq z \bullet \log_y x$
    III. $\lfloor \pi^e \rfloor = 22$ false
    IV. $\log_x a + \log_x b + \log_x c = \frac{ab}{c}$ false **C**

22. $12 + 48 = 60$ **B**

23. $(1+i)^{40} = (1+i)^{2^{20}} = (2i)^{20} = 2^{20}$ **A**

24. $x = \frac{ky}{z^2} \qquad 9 = \frac{3k}{4} \quad k = 12 \qquad 16 = \frac{12y}{6} \qquad y = 8$ **C**

25. 2x6 A

26. $\frac{5}{x^2 + 6x + 9} \geq 0 \qquad \frac{5}{(x+3)(x+3)} \geq 0 \quad x \neq -3$ all other x satisfy the equation **E**

27. $V = (3, 2)$ ordinate is the y value the ordinate is 2 **D**

28. $(x+4)^{22} \quad \frac{4^3 \cdot 1540 + 4^5 \cdot 26334}{2} = 13532288$ **C**

29. I. True
    II. False
    III. False
    IV. False **E**

30. $(x+1)^3 = 9x^2 + 22x - 83 \qquad (x+1)(x+1) = (x^2 + 2x + 1)(x+1) = x^3 + 3x^2 + 3x + 1 = 9x^2 + 22x - 83$
    $x^3 - 6x^2 - 19x + 84 = 0$
    3] 1 -6 -19 84 — $x^2 - 3x - 28 = 0 \qquad \frac{3 \pm \sqrt{9 + 4\cdot 28}}{2} = \frac{3 \pm 11}{2} = 7, -4$
    3 -9 -84
    1 -3 -28 0 — But -4 does not work in the original equation. $3 \cdot 7 = 21$ **D**