(10)

Calculus Individual Test Sunset Invitational January 26, 2002

1 If $f(x)=2x^3+\sqrt{\pi x}+\sin x+\pi^{\pi}$ then $\frac{df}{dx}$ evaluated at π is

A) $\frac{12\pi^3-2\pi+1}{2\pi}$ B) $6\pi^2-2$ C) $6\pi^2+\frac{1}{2\pi}-1+\pi^{\pi}$ D) $6\pi^2+\frac{1}{2\pi}+\pi\cdot\pi^{\pi-1}$ E) NOTA

2 Let $f(x)$ be a polynomial function such that $f(-1)=3, f(-2)=5, f(-3)=2, f'(-2)=0$, and $f''(-2)=3$. The point $(-2,5)$ is a

A) Relative maximum B) Relative minimum C) Point of Inflection
D) Cusp E) NOTA

3 Find $\lim_{x\to\infty}\frac{\sqrt{4x^2-1}}{x^2}$

A) 0 B) 2 C) 4 D) ∞ E) NOTA

4 Evaluate $\lim_{x\to\infty}\frac{e^{\frac{1}{x}}+e^{\frac{2}{x}}+e^{\frac{3}{x}}+\mathbf{L}+e}{x}$

A) 0 B) $e-1$ C) e D) $e+1$ E) NOTA

5 If $[x]$ represents the greatest integer value which is less than or equal to x. Then evaluate $\frac{1}{2}\cdot\int_0^{\pi}[\sin x]dx$

A) 1 B) 2 C) $\frac{\pi}{2}$ D) π E) NOTA

6 If $f=\cos^2(x^2+3)+\sin^2(x^2+3)+e^{\pi}$ Find $f'(x)$

A) $2x\cos(x^2+3)\sin(x^2+3)+2x\sin(x^2+3)\cos(x^2+3)$

B) $2x\cos(x^2+3)\sin(x^2+3)+2x\sin(x^2+3)\cos(x^2+3)+\pi e^{\pi-1}$

C) $4x\sin(x^2+3)\cos(x^2+3)+\pi e^{\pi-1}$

D) $\pi e^{\pi-1}$

E) NOTA

7 Evaluate $\int_0^4\sqrt{16-x^2}dx$.

A) $\frac{\pi}{2}$ B) π C) 2π D) 4π E) NOTA

8 A ladder is to reach over a fence $\frac{27}{8}$ meters high to a wall 8 meters behind the fence. Find the length of the shortest ladder that can be used.

A) $\frac{64}{4}$ B) $\frac{91\sqrt{2}}{8}$ C) $\frac{45\sqrt{2}}{4}$ D) $\frac{125}{8}$ E) NOTA

9 Evaluate $\lim_{x\to 0}\sqrt{x}$

A) 0 B) 1 C) 2 D) 3 E) NOTA

10 If $\lim_{x\to\infty}\sqrt{Ax^2+Bx}-x=2$, then find $A+B$

A) 5 B) 13 C) 17 D) Does not exist E) NOTA

11 $\lim_{x\to 0}\frac{1}{x}\left(e^{1+x}-e\right)$

A) -1 B) 0 C) 1 D) e E) NOTA

12 $\int_{\pi}^{\pi}\frac{x+\sin x}{x^2-\cos x}dx$

A) $-\pi$ B) 0 C) π D) $\frac{\sqrt{3}}{2\pi}$ E) NOTA

13 $\lim_{x\to 2}2x+\pi=4+\pi$. The $\in/\partial$ definition of limit requires that for each $\in>0$, a ∂ must exist. Of the following choices, which value would be the largest value for ∂ which would satisfy the definition when $\in=.1$?

A) .04 B) .05 C) $\frac{.04}{\pi}$ D) $.05+\pi$ E) NOTA

14 What is the slope of the normal line to the graph of $\text{Arctan}(x)$ when $x=\frac{\pi}{2}$?

A) $\frac{1}{1+\pi}$ B) $-\left(4+\pi^2\right)$ C) slope is undefined

D) Normal line doesn't exist E) NOTA

15 Evaluate $\int_{0.0135}^{0.\overline{027}}dx$

A) $\frac{1}{74}$ B) $\frac{1}{37}$ C) 0.0135 D) $\frac{29}{1998}$ E) NOTA

16 Evaluate $\int_{-1}^{1}\frac{1}{x}dx$

A) 0 B) $\frac{1}{2}$ C) 1 D) e E) NOTA

17 $\int \frac{dx}{x\ln(x^2)}$ for x > 10

A) $\frac{1}{2}\ln(\ln x)+C$ B) $\ln(\ln x^2)+C$ C) $\frac{1}{2\ln x}+C$

D) $\frac{1}{e^x \ln x}+C$ E) NOTA

18 Let $f(x)=\int_{\frac{\pi}{2}}^{x^2-2x+8} \sin^4(x^6)\,dx$

Find $f'(-1)$

A) -1 B) 0 C) 1 D) $\sin^4 1$ E) NOTA

19 Let f be the function defined on the interval $I=(0,1)$ as follows:

$$f(x)=\begin{cases}0 & x \text{ is irrational}\\ \frac{1}{n} & \text{if } x=\frac{m}{n}\end{cases}$$

(with $\frac{m}{n}$ in lowest terms, m and n are integers with n > 0)

Then $f(x)$

A) is continuous on I B) is discontinuous on I

C) is continuous at every rational point in I D) is continuous at every irrational point in I

E) NOTA

20 Consider a function f which is continuous on the real numbers. If there is a value x, such that $x<0$, $f(x)<0$ and $f(x)=-f(-x)$ then $f(x)$

A) is symmetric about the origin B) is symmetric about the y-axis.

C) is strictly increasing. D) has a solution s, such that $s>x$

E) NOTA

21 Approximate the value of $e^{0.1}$ using differentials

A) 0.1 B) 1.1 C) 1.105 D) e E) NOTA

22 The sum of two nonnegative numbers, x and y, is 12. What is the largest possible product of x^2 and y?

A) 0 B) 8 C) 36 D) 256 E) NOTA

23 Using four rectangles on a regular partition of [0,2], calculate the **lower sum** approximation of $\int_0^2 (x^2+1)\,dx$

A) 1 B) $\frac{14}{3}$ C) $\frac{9}{4}$ D) $\frac{15}{4}$ E) NOTA

24 Evaluate:

$$\lim_{x\to 0}\frac{10^x-1}{x}$$

A) $\ln(10)$ B) $\ln(x)$ C) e^x D) 10 E) NOTA

25 $$\int_0^3 \frac{x}{\left(x^2-1\right)^{\frac{2}{3}}}dx$$

A) $\frac{-27}{2}$ B) $\frac{9}{2}$ C) 3 D) DNE E) NOTA

26 Let $[x]$ denote the greatest integer less than or equal to x. If n is a positive integer, then

$$\lim_{x\to -n^-}(|x|-[x])-\lim_{x\to n^-}(|x|-[x])=$$

A) -2 B) 0 C) 2 D) 2n-1 E) NOTA

27 The curve whose equation is $2x^3+3x-2xy-y=6$, has two asymptotes. Identify those lines.

A) x = -1 and y = -2 B) x = -2 and y = 1 C) $x=\frac{1}{2}$ and y = x

D) $x=-\frac{1}{2}$ and y = x + 1 E) NOTA

28 For what values of m and b will the following function have a derivative for every x?

$$f(x)=\begin{cases}x^2+x-3 & x\le 1\\ mx+b & x>1\end{cases}$$

A) m = 3, b = -4 B) m = -2, b = -3 C) m = 1, b = -4
D) m = -2, b = 1 E) NOTA

29 Let $f(x)$ be continuous on a bounded interval $[a,b]$, where $a\neq b$, such that $f(a)=1$ and $f(b)=3$, and $f'(x)$ exists for every x in (a,b). What does the mean-value theorem say about f?

A) There exists a number c in the interval (a,b) such that $f'(c)=0$

B) There exists a number c in the interval (a,b) such that $f'(c)=2$

C) There exists a number c in the interval (a,b) such that $f'(c)=2(b-a)$

D) There exists a number c in the interval (a,b) such that $(b-a)f'(c)=2$

E) NOTA

30 The radius of a circle is decreasing at a rate of 0.5 cm per second. At what rate, in cm^2 / sec, is the circle's area decreasing when the radius is 4 cm?

A) 4π B) 2π C) π D) $\frac{\pi}{2}$ E) NOTA