

1

A $f(x) = 2x^3 + \sqrt{\pi x} + \sin x + \pi^x$

$$6x^2 + \frac{1}{2}(\pi x)^{-\frac{1}{2}} \pi + \cos x$$

$$6\pi^2 + \frac{1}{2}(\pi\pi)^{-\frac{1}{2}} \pi + \cos \pi = 6\pi^2 + \frac{1}{2\pi} - 1 = \frac{12\pi^3 - 2\pi + 1}{2\pi}$$

2

B Since f is a polynomial it is continuous and differentiable. $f(-2) = 0$ means that it is a critical point. $f'(-2) = 3 > 0$ means concave up. so local minimum.

$$f(-3) =$$

3

A

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 1}}{x^2} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \sqrt{4x^2 - 1}}{\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{4x^2 - 1}{x^2}}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{1}{x^2}}}{\frac{1}{x}} = \frac{2}{\infty} = 0$$

4

B

$$\lim_{x \rightarrow \infty} \sum_{n=1}^x e^{\frac{1}{n}} \cdot \frac{1}{n} = \int_0^1 e^x dx = e^x - 1 \Big|_0^1 = e - 1 \quad \text{B) } e - 1$$

5

E

$\frac{1}{2} \cdot \int_0^{\pi} [\sin x] dx$ Since $\sin x$ only reaches the value of 1 at $\frac{\pi}{2}$, the function reduces to 0 for all other points. Integral is 0 E) NOTA

6

E

$\cos^2 x + \sin^2 x = 1$ The derivative of a constant is 0

7

D

The function is a quarter circle with radius 4 $\frac{16\pi}{4} = 4\pi$

8

D

$$\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(\left(\frac{27}{8}\right)^{\frac{2}{3}} + (8)^{\frac{2}{3}}\right)^{\frac{3}{2}} = \frac{125}{8}$$

9

E

The limit does not exist at 0, since the function does not exist for negative x

10

A

$$\lim_{x \rightarrow \infty} \frac{\sqrt{Ax^2 + Bx} - x \cdot \frac{\sqrt{Ax^2 + Bx} + x}{\sqrt{Ax^2 + Bx} + x}}{\sqrt{Ax^2 + Bx} + x} = 2$$

$$\lim_{x \rightarrow \infty} \frac{Ax^2 + Bx - x^2}{\sqrt{Ax^2 + Bx} + x} = 2 \quad \text{normally, } Ax^2 - x^2 \text{ would dominate and the limit would be}$$

infinity. Since the limit is 2, $Ax^2 - x^2 = 0 \rightarrow A = 1$

$$\lim_{x \rightarrow \infty} \frac{Bx}{\sqrt{x^2 + Bx} + x} = 2 \quad \text{means } B=2 \quad \text{Thus } A^2 + B^2 = 1 + 4 = 5$$

11

D

this is the definition of the derivative of e^x evaluated at $x=1$. D) e

12

B

Function is an odd function and is thus symmetric through the origin. The integral is 0

13

A

vertical translations do not effect the value of ∂ , so just consider the function $2x$. ∂

$$\text{must be less than } \frac{\epsilon}{|2|} = .05 \text{ so } .04$$

- 14 E slope of tangent line is $\frac{1}{1+x^2} = \frac{1}{1+\left(\frac{\pi}{2}\right)^2} = \frac{1}{\frac{4+\pi^2}{4}} = \frac{4}{4+\pi^2}$ Normal is $-\frac{4+\pi^2}{4}$
- 15 A $\int_{0.0135}^{0.027} dx = \int_{\frac{1}{74}}^{\frac{1}{37}} dx = \frac{1}{37} - \frac{1}{74} = \frac{1}{74}$
- 16 E function has singularity at 0. No real number answer E
- 17 A The apparent answer $\frac{1}{2} \ln(\ln x^2) + C$ does not appear, so it is tempting to choose E without further consideration. In fact, answer A differs by the apparent answer by a constant.
- $$\frac{1}{2} \ln(\ln x^2) + C = \frac{1}{2} [\ln(2 \ln x)] + C = \frac{1}{2} \ln 2 + \frac{1}{2} \ln(\ln x) + C = \frac{1}{2} \ln(\ln x) + \left(C + \frac{1}{2} \ln 2 \right)$$
- 18 B In general if $F(x) = \int_a^{g(x)} f(t) dt$ then $F'(x) = f(g(x))g'(x)$. Since
- $$g(x) = x^2 - 2x + 8$$
- $$g'(x) = 2x - 2$$
- $$g'(-1) = 0$$
- so the answer is 0
- 19 D Let a be an irrational number in I , and let $\epsilon > 0$ be given. Then, in an open interval centered at a and contained within I , there are less than $1+2+\dots+(q-1) = q(q-1)/2$ positive rational numbers less than 1 and of the form m/n , where m and n are positive integers and n is less than q . From this finite list of rational numbers, choose the one, r , that is closest to a and let $\delta = |r - a|$. Then, within the open interval $(a-r, a+r)$, the value of $f(x)$ will be less than $1/q$, so $f(x)$ will be less than ϵ . f is continuous at every irrational
- 20 D If the value of $f(x)$ changes sign between a and b , then there is a zero between a and b .
- 21 B C is the actual answer using a calculator, but we need the value of the approximation so $y = e^x \rightarrow y' = e^x \quad dy = y'(0) * dx + e^0 \rightarrow 1 * .1 + 1 = 1.1$
- 22 D $x + y = 12 \rightarrow y = 12 - x$
- $$M = x^2 y \rightarrow M = x^2 (12 - x) = 12x^2 - x^3$$
- $$M' = 24x - 3x^2 \rightarrow x = 0, 8$$
- Check 0, 8, and 12 $\rightarrow 0, 256, 0$ Answer is 256 D
- 23 D The regular partition will divide $[0, 2]$ into 4 intervals of length $1/2$ at 0, $1/2$, 1, and $3/2$. The heights $(x^2 + 1)$ will be 1, $5/4$, 2, $13/4$. so the area is $1/2(1 + 5/4 + 2 + 13/4) = (1/2)(30/4) = 15/4$
- 24 A $\lim_{x \rightarrow 0} \frac{10^x - 1}{x}$ looks like the definition of derivative for $y = 10^x$ when $x = 0$. (use h for x in the problem $\lim_{h \rightarrow 0} \frac{10^{(0+h)} - 10^0}{h}$)
- 25 B The integrand goes to infinity when $x=1$ so it is necessary to use improper integrals

$$\lim_{N \rightarrow 1} \int_0^N \frac{xdx}{(x^2-1)^{\frac{2}{3}}} + \int_N^3 \frac{xdx}{(x^2-1)^{\frac{2}{3}}} = \lim_{N \rightarrow 1^-} \left[\frac{3}{2}(x^2-1)^{\frac{1}{3}} \right]_0^N + \lim_{N \rightarrow 1^+} \left[\frac{3}{2}(x^2-1)^{\frac{1}{3}} \right]_N^3 = \frac{3}{2} + 3 = \frac{9}{2}$$

26 C The $\lim_{x \rightarrow n} [x] = n-1$, $\lim_{x \rightarrow n} |x| = n$ so

$$\lim_{x \rightarrow n^-} (|x| - [x]) - \lim_{x \rightarrow n^+} (|x| - [x]) = n - (-(n-1)) - n - (n-1) = 2n+1-2n+1=2$$

27 D Solve for y: $y = \frac{2x^2 + 3x - 6}{2x+1} \rightarrow y = x+1 + \frac{-7}{2x+1}$

The first form is undefined when $x = -1/2$

The limit as $x \rightarrow$ infinity in the second form is $y = x+1$

28 A $f(1) = 1+1+3 = -1$

$$f'(1) = 2x+1 = 3$$

$$mx + b = -1$$

$$m = 3$$

$$\text{so } 3*1+b=-1 \quad b = -4$$

29 D there is a point c, where the $f'(c)$ is equal to the slope of the secant line from a, $f(a)$ to

$$b, f(b) \quad f'(c) = \frac{f(b) - f(a)}{b-a} \rightarrow (b-a) f'(c) = 3-1=2$$

30 A $A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi 4 \frac{1}{2} = 4\pi$