(15)

Calculus Individual Answers

1	D		11	E		21	C
2	C		12	A		22	A
3	D		13	C		23	B
4	B		14	B		24	B
5	C		15	B		25	C
6	B		16	D		26	A
7	D		17	C		27	B
8	B		18	D		28	C
9	E		19	B		29	D
10	D		20	C		30	B

1. Use first derivative function from calculator. Answer is 80.530
A) made up distractor
B) is the function value at 1.1
C) is the integral value from 0 to 1.1
D) correct answer

2. Use logarithmic differentiation

$$y = x^x \to \ln y = x\ln x \to \frac{1}{y}y'$$

$$= \ln x + x\cdot\frac{1}{x} \to y' = x^x(\ln x + 1)$$

A: Used the power rule
B: Forgot to multiply through by y to solve for y'
C: Correct Answer
D: Confused adding a constant from rules for Integration

3. By division

$$f(x) = \frac{3x^3+3x+1}{x^2+\frac{1}{3}x+1} \to f(x) = 3x - 1 + \frac{\frac{x}{3}+2}{x^2+\frac{1}{3}x+1}$$

As x gets larger, the fractional part approaches 0, so the function approaches $f(x) = 3x - 1$.
A: Uses only the numerator
B: Uses only the numerator
C: Has a close values only for x's near 10000.
D: Correct answer

4. Take the limit. The function has a "hole" for $x = \frac{2}{3}$ so you cannot evaluate f(x) there.

The function simplifies to $\frac{(3x-5)(3x-7)}{3(x-3)(3x-11)}$. The value of this for $x = \frac{2}{3}$ is $\frac{5}{21}$. Use POLY to factor polynomials
A) Tries to take limit by substitution, and gets a 0 in the numerator
B) The correct answer
C) Tries to take limit by substitution, and gets a 0 in the denominator, thinks $\frac{1}{x^2}$
D) Tries to take limit by substitution, and gets a 0 in the denominator, thinks $\frac{1}{x}$

5. An example is $f(x) = \left[\frac{x}{2}\right]$. [] is the greatest integer function
A) I is FALSE because you can only conclude that it is continuous AT x = 0.
B) II is FALSE, there is no mention of derivatives.
C) is the correct answer. III is TRUE, to have a limit, the function must be defined on an interval.
D: I is false

6. Evaluate the limits of continuous functions by calculating

$$f\left(\lim_{x\to -1} g(x)\right) = f(-1+3) = f(2) = 4$$

A) Evaluates inside function only
B) correct answer
C) ?
D) does not apply limit theorem for continuous functions
E) does not calculate interior limit correctly

7. To make the function continuous the value of the function must match for the top and middle parts and for the middle and bottom parts.

$$\frac{x^3+4x^2+x-6}{x^2+x-2} = \frac{(x+3)(x+2)(x-1)}{(x+2)(x-1)} = x+3 \text{ when } x \neq 3$$

x+a must equal x+3 when x = 1 so a=3

and x+3 must equal $\frac{x}{3}+b$ when x=3, so b=5

Answer is D) 8

8. If the two graphs cross the x-axis at $(a,0)$ then their respective tangent lines are:
$f'(a)(x-a)$ and $g'(a)(x-a)$.
The quotient would be

$$\frac{f(x)}{g(x)} \approx \frac{f'(a)(x-a)}{g'(a)(x-a)} \text{ so}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(a)(x-a)}{g'(a)(x-a)} = \lim_{x\to a}\frac{f'(a)}{g'(a)}$$

This is a geometric "proof" of L' hopital's rule
Answer is B

9. The two functions provided have THREE points of intersection. Two are obvious from the display of a graphing calculator. The third exists because we know that exponential function with base greater than 1 "grow" faster than x^2.
A: Student assumed that the intersection points occur symmetric to the y-axis
B: The leftmost x-intercept.
C: The x-intercept of the middle point.
D: The sum of the slopes at A and B
E: Correct answer. The tangent line at the third point is close to vertical

10. $a = 6t - 18 \Rightarrow v = 3t^2 - 18t + v_0$ since $v_0 = 24$ we get $v = 3t^2 - 18t + 24 \Rightarrow s = t^3 - 9t^2 + 24t + s_0$
Substitute $s_1 = 20$ $20 = 1^3 - 9\cdot 1^2 + 24\cdot 1 + s_0 \Rightarrow s_0 = 4$
Factoring v tells us that there are "stop" points at t=2 and t=4.
total distance traveled from 0 to 3 is

$$|x(2)-x(0)| + |x(3)-x(2)|$$

$$= |24-4| + |22-24| = 20 + 4 = 24$$

D is the correct solution.

11. The DEFINITE integral is a constant. The derivative of this constant is 0
A) The value of the definite integral
B) Fundamental theorem IF boundaries a to x.
C) The INDEFINITE integral (without taking the derivative)
D) The opposite of answer B as a distractor
E) Correct Answer

12. The table is of the bird's VELOCITY. Notice that not all graphs have the same SCALE.
Graph B's matches the data given. Since we are looking for position, we need B's antiderivative. Sign changes +/- near 0 which indicates a MAX in the same location. Sign change -/+ near one indicates a MIN.
+/- less than 2 and -/+ greater than 2 MAX then Min. +/- near three MAX
Answer is A ~~E~~ E

13. $\lim_{x\to\infty}\sqrt{ax^2+bx}-cx=2$
Solve this by multiplying by

$$\lim_{x\to\infty}\left(\sqrt{ax^2+bx}-cx\right)\left(\frac{\sqrt{ax^2+bx}+cx}{\sqrt{ax^2+bx}+cx}\right)$$

$$=\lim_{x\to\infty}\frac{\left(ax^2+bx\right)-c^2x^2}{\sqrt{ax^2+bx+cx}}$$

We can see that the coefficient of the x^2 term in the numerator MUST be zero or the limit would be ∞ and not 2.
Thus $a-c^2=0\Rightarrow a=c^2$
This plus the fact that $a+c^2=8$ means that
$2a=8\Rightarrow a=4$ and $c=2$

Reducing the above limit we get

$$\lim_{x\to\infty}\frac{\left(ax^2+bx\right)-c^2x^2}{\sqrt{ax^2+bx+cx}}=\lim_{x\to\infty}\frac{bx}{\sqrt{ax^2+bx}+cx}$$

$$\Rightarrow\frac{b}{\sqrt{a}+c}=2$$

so $\frac{b}{4}=2\Rightarrow b=8$. Therefore $a+b+c=4+8+2=14$
A) Gets confused with $\infty-\infty$. thinks that it is ∞
B) Last year's answer
C) Correct Answer
D) Gets confused with $\infty-\infty$. thinks that it is 0

14. Let a circle represent the area of the town. Then the population funtion would create a 3-D graph. The population of this graph is the volume, use cyllindrical shells.
The answer is B

15. $y=x^2\sin y$ by implicit differentiation
$y'=2x\sin y+x^2(\cos y)y'$ Solving for

$$y'=\frac{2x\sin y}{1-x^2\cos y}=\frac{x}{x}\cdot\frac{2x\sin y}{1-x^2\cos y}=\frac{2\left(x^2\sin y\right)}{x\left(1-x^2\cos y\right)}$$

Since $y=x^2\sin y$ replace $x^2\sin y$ in the numerator with y.
Answer is $\frac{2y}{x\left(1-x^2\cos y\right)}$ B
A) The result of the differentiation with - replaced with +
B) The correct answer
C) improper use of the product rule
D) improper use of the product rule

16. I. f(2x) is compressed so that the maximums would occur at x = 1 and x = 2 FALSE
II. Squaring the function squares the max values for x>1 but does NOT change the minimum at 8 into a maximum. the 100 value is still a minimum FALSE
III. f(2x) has a rel max at 2 and a <u>higher value</u> rel min at 4. There must be another pair of "humps" between the two values. TRUE
III only D

17. $\int\frac{\ln^2 x}{x}dx$ let $u=\ln x$ so $\frac{du}{dx}=\frac{1}{x}$ so change the variables and get $\int u^2du$, so the antiderivative is $\frac{1}{3}u^3+C$ thus I and II are the answers.
C is the correct solution.

18. x^2 is differentialble at 0, and has derivative $2x$ so that for values near 0, the graph is indistinguishable from $y=1$. $|x|$ never loses it's "corner" at any magnification.

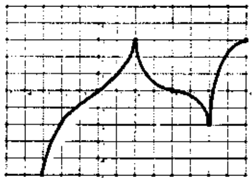
19. This is the definition of the derivative for $y=8x^8$ evaluated at $x=\frac{1}{2}$ so

$$y'=64x^7\Rightarrow y'\left(\frac{1}{2}\right)=64\left(\frac{1}{2}\right)^7=\frac{1}{2}$$

Answer is B

20. Average value is $\frac{\int_1^4 14\pi x^2dx}{4-1}=98\pi$
C is the correct solution

21.



22. Use the Fundamental Theorem of Calculus by evaluating the integrand at the boundaries of the integral. Plug in the boundaries to tan(x)

Tan(B) - Tan(A) = $(2x-3)\tan(x^2-3x)-\frac{\tan\sqrt{x}}{2\sqrt{x}}$

A) correct answer
B) is the integral
C) derivative of the integrand
D) the proper answer but then evaluated at 1

23. The general antiderivative is x^3+3x^2+3x+C
An example of an antiderivative is when C = 1 thus

$x^3+3x^2+3x+1=(x+1)^3$

Thus any other constant C could be broken down into two parts: 1 and C
A) "almost" the antiderivative. The - sign makes it wrong
B) Correct Answer
C) This is the derivative
D) This is the antiderivative of the antiderivative

24. The rate of change of r^3 is $\frac{dr^3}{dt}=3r^2\frac{dr}{dt}$

if this rate is 12 times that of r then, $12=3r^2$.
So $r=2$
Answer is B

25. To calculate the region, draw the graphs, Use ISECT to find x value for the first instersection. Store the value of x into A.
A = .785398163397
Use ISECT again to find the second intersection. Store x into B.
B = 3.92699081699
Calculate
fnInt(sin x- cos x, x, A, B) = 2.82842712475

or $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\cos x-\sin x)dx=2\sqrt{2}$

Correct answer is C or E

26. The function factors as follows.

$$y=\frac{(x-2)\left(x-\frac{1}{3}\right)(x-4)}{\left(x-\frac{1}{3}\right)(x-3)}$$

The domain is all reals except $\frac{1}{3}$ and 3.

y has a REMOVABLE discontinuity at $x=\frac{1}{3}$ and a vertical assymptote at x = 3.
y has NO horizontal assymptotes, but does have an oblique one.
Answer is A (I only)

27. At x = 1, $y+1\sqrt{1+\frac{1}{1}}=\sqrt{2}$

$$y'=x\cdot\frac{1-\frac{1}{x^2}}{2\sqrt{x+\frac{1}{x}}}+\sqrt{x+\frac{1}{x}}$$

at x = 1, $y'=0+\sqrt{1+\frac{1}{1}}=\sqrt{2}$

Tangent line: $y-\sqrt{2}=\sqrt{2}(x-1)$

$y=\sqrt{2}x-\sqrt{2}+\sqrt{2}$ $\quad y=\sqrt{2}x$

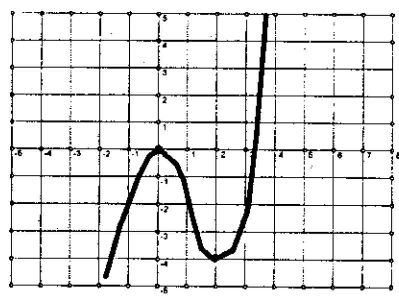
Answer is B

28. C. Take the derivatives

$y=x^3+2x^2+\sin x+e^{2x}+x+1$

$$y'=3x^2+4x+\cos x+2e^{2x}+1$$

29. If $y=x^3-3x^2$, then $y'=3x^2-6x$ has critical points x=0, x=2.
From the graph we can see that the rel max at 0 will only



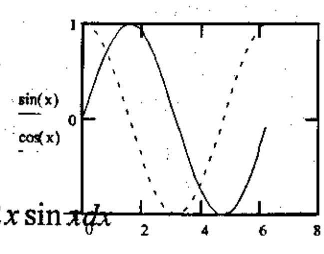
contribute 1 zero unless we translate it up. Therefore $k>0$. The relative min at (2,4) will contribute 2 zeros until that point is translated up to 0. Thus $k<4$

Answer $0<k<4$. Answer is D

30. Use integration by parts

$\int udv=uv-\int vdu$

$u=x^2$ $\quad v=\sin x$
$du=2xdx$ $\quad dv=\cos xdx$



$\int x^2\cos xdx=x^2\sin x-\int 2x\sin xdx$

so $f(x)=x^2\sin x+C$
B