Calc Team

Answer for MIAMI SPRINGS SENIOR HIGH SCHOOL MATHEMATICS INVITATIONAL on APRIL 16, 1994

Q	Answer	Q	Answer	Q	
1	EDCBA	6	$x_5 = 1.4967$	11	A= 2π; B=$\frac{14\pi}{9}$ **answer =** $\frac{9}{7}$
2	π	7	$\pi^{\pi}(\ln\pi + 1)$	12	$\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$
3	-3	8	$80{,}000\sqrt{10}$	13	$-\frac{1}{5}\left(\frac{\sqrt{1-x^2}}{x}\right)^5 + C$ or $-\frac{(1-x^2)^{\frac{5}{2}}}{5x^2} + C$
4	4	9	514	14	$\frac{8}{3}\sqrt{2}$
5	$\frac{5}{4}$	10	A=3; B=6 **answer = 27**	15	50

<u>Calculus Team Solutions</u>

① E D C B A

② $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} dx = x\Big|_{\pi/2}^{3\pi/2} = \pi$

③ $u = \sqrt{3x^2+1} \quad u(1) = 2$

$u' = \frac{1}{2}(3x^2+1)6x = \frac{3x}{\sqrt{3x^2+1}}$

$u'(1) = \frac{3}{2}$

$f(u) = \frac{u+1}{u-1} \quad f'(u) = \frac{-2}{(u-1)^2}$

$[f(u)]' = f'(u)\cdot u' = -2\cdot 3/2 = -3$

④ $\lim_{n\to\infty} \frac{16}{n}\left(\frac{1}{n^3} + \frac{8}{n^3} + \frac{27}{n^3} + \ldots + 1\right)$

$\lim_{n\to\infty} \frac{2}{n}\left(\frac{8}{n^3} + \frac{8\cdot 8}{n^3} + \frac{8\cdot 27}{n^3} + \ldots + 1\right)$

$\lim_{n\to\infty} \frac{2}{n}\left[\left(\frac{2\cdot 1}{n}\right)^3 + \left(\frac{2\cdot 2}{n}\right)^3 + \left(\frac{2\cdot 3}{n}\right)^3 + \ldots 1\right]$

$\Delta x = \frac{2}{n} = \frac{b-a}{n} = \frac{2-0}{n}$

$w_i = 0 + dx = i\,\frac{2}{n} \Rightarrow \int_0^2 x^3\,dx = \frac{1}{4}x^4\Big|_0^2 = 4$

⑤ $y = \frac{(2x-3)^2(3x-2)^3}{(x^3-3)^3(3x^3-2)^2} \quad y(1) = \frac{-5^3}{-8(-3)^2} = \frac{5}{8}$

$\ln y = 2\ln(2x-3) + 3\ln(3x-2) - 3\ln(x^2-3) - 2\ln(3x^3-2)$

$\frac{1}{y}y' = \frac{4}{2x-3} + \frac{9}{3x-2} - \frac{6x}{x^2-3} - \frac{2}{3x^3-2} \quad 9x^2$

$y' = \frac{5}{8}\left(\frac{4}{-5} + \frac{9}{-5} + \frac{6}{-2} - \frac{18}{-5}\right)$

$\frac{5}{8}\left(-\frac{20}{10}\right) = -\frac{5}{4}$

⑥ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1$

$x_2 = 2.3833$

$x_3 = 1.736224$

$x_4 = 1.5303703$

$x_5 = 1.496735$

⑦ $x^x + 2\cos^2 x + 2\tan^2 x + \cos 2x - 2\sec^2 x + e^\pi + \frac{1}{2}$

Simplify to

$y = x^x + 4\cos^2 x - \frac{5}{2} + e^\pi$

$y' = x\ (\ln x + 1) - 8\cos x \sin x$

$\pi^\pi(\ln\pi + 1) - 0$

$\pi^\pi(\ln\pi + 1)$

⑧ $S^2 = 12000 - 2x$

$S = \sqrt{12000 - 2x}$

domain $0 \le x < 6000$

$R = x\sqrt{12000 - 2x}$

$R' = \sqrt{12000-2x} + x\cdot\frac{1}{2}(12000-2x)^{-1/2} - 2x$

$\sqrt{12000-2x} + \frac{-x}{\sqrt{12000-2x}}$

$\frac{12000 - 2x - x}{\sqrt{12000-2x}}$

$R' = \frac{12000-3x}{\sqrt{12000-3x}} = 0$

Critical $\Rightarrow x = 4000$

$R = 4000\sqrt{4000} = 80{,}000\sqrt{10}$

⑨ $f(2) = 2^{3^2} = 2^9 = 512$ A *

$f(1) = 1^{2^1} = 1$

$y = x^{x+1}$ $\quad A + B + C\ D = 514$

$\ln y = x(x+1)\ln x$

$\frac{1}{y}y' = (x+1)\ln x + x\ln x + \frac{x(x+1)}{x}$

$y' = y\left[(x+1)\ln x + x\ln x + \frac{x(x+1)}{x}\right]$

$y'(1) = 1\ \left(2\cdot 0 + 1\cdot 0 + \frac{1(1+1)}{1}\right)$

$y'(1) = 2$ B *

* C = 0

* D = Some number

⑩ $f(x) = 3\sqrt{x+1}$

$\int_{-1}^{8} 3\sqrt{x+1}\,dx$

$u = x+1$

$du = dx$

$3\int_0^9 u^{1/2}\,du = 3\,{}^2\!/\!_3\,u^{3/2}\Big|_0^9 = 54$

Ⓐ $54 \div 9 = 6$ $\quad$ Ⓑ $54 \div 9 = 6$

$3\sqrt{x+1} = 6 \Rightarrow \sqrt{x+1} = 2$

$x+1 = 4$

$x = 3$

$c = \int_3^3 \text{etc} = 0$

$A + B + AB^{C+1}$

$3 + 6 + 3\cdot 6^1$

$9 + 18 = 27$

(11) $N_1 = \pi \int_0^1 (3x^2+1)\,dx$

$x^3 + x \Big|_0^1 = 2\pi$

$V_2 = \frac{2\pi}{6} \int_0^1 6x\sqrt{3x^2+1}\,dx$

$u = 3x^2+1$
$du = 6x\,dx$

$\frac{\pi}{3} \int_1^4 u^{1/2}\,du$

$\frac{\pi}{3} \cdot \frac{2}{3} u^{3/2} \Big|_1^4$

$\frac{2\pi}{9}(8-1) \Rightarrow \frac{14\pi}{9}$

ratio $= 2\pi \cdot \frac{9}{14\pi} = \frac{9}{7}$

(12) $\int \cos^3 x \sin^4 x\,dx$

$\int \cos^2 x \sin^4 x \cos x\,dx$

$\int (1-\sin^2 x)\sin^4 x \cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int (1-u^2)u^4\,du$

$\int u^4 - u^6\,du$

$\frac{1}{5}u^5 - \frac{1}{7}u^7$

$\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$

(13) $\int \frac{(1-x^2)^{3/2}}{x^6}\,dx$

$x = \sin\theta \qquad dx = \cos\theta\,d\theta$

$1 - x^2 = 1 - \sin^2\theta = \cos^2\theta$

$\int \frac{(\cos^2\theta)^{3/2}}{\sin^6\theta}\cos\theta\,d\theta$

$\int \frac{\cos^4\theta}{\sin^6\theta}\,d\theta$

$\int \frac{\cos^4\theta}{\sin^4\theta} \cdot \frac{1}{\sin^2\theta}\,d\theta$

$\int \cot^4\theta \cdot \csc^2\theta\,d\theta$

$-\frac{1}{5}\cot^5\theta + C$

(14) $v = kt \qquad v = 2t \qquad a = 2$

$2 = k \cdot 1 \qquad s = t^2$

$k = 2$

$t^2 = 2 \Rightarrow t = \pm\sqrt{2}$

$2\int_0^{\sqrt{2}} 2 - t^2\,dt = 2\left(2t - \frac{1}{3}t^3\right)\Big|_0^{\sqrt{2}}$

$2\left(2\sqrt{2} - \frac{1}{3}(\sqrt{2})^3\right) = \frac{8}{3}\sqrt{2}$

(15) $\iiint (\quad)\,dx\,dy\,dz$

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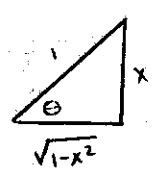

$\sin\theta = x$

$\therefore \cot\theta = \frac{\sqrt{1-x^2}}{x}$

$-\frac{1}{5}\left(\frac{\sqrt{1-x^2}}{x}\right)^5 + C$

$-\frac{(1-x^2)^{3/2}}{5x^5} + C$