January 2001 Invitational
Pre-Calculus Test Answers

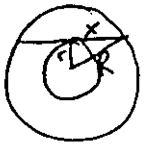
Individual

1. C
2. C
3. C
4. D
5. A
6. D
7. D
8. A
9. A
10. B
11. E
12. B
13. A
14. D
15. A
16. A
17. A
18. C
19. C
20. B
21. E
22. A
23. C
24. B
25. B
26. B
27. B
28. C
29. E
30. D

Team

1. 30.1
2. 80
3. 9.5
4. 9
5. 1.22
6. $2^{\frac{3}{4}}$
7. $x = 4$
8. 21
9. 30
10. 2.4, 12/5
11. 93
12. Ace of Clubs
13. $h = 5$
14. 15
15. F

(22)



$x = 4.35$

$R^2 = r^2 + 4.35^2$

$\text{Area} = \pi(R^2 - r^2)$

$= \pi \cdot 4.35^2$ $\boxed{A}$

≈ 59.4

(23) If flipped n times, there are 2^n possibilities. Let $A(n)$ be the number of sequences w/no consecutive occurences of H. $A(1)=2$, $A(2)=3$, $A(3)=5$. Follows Fibonacci sequence, so $A(10)=144$

$\frac{144}{1024} = \frac{9}{64}$ $\boxed{C}$

(24) take log base 7 of each side

$\frac{1}{2} + \log_7 x \cdot \log_7 x = 7\log_7 x$

$u = \log_7 x$, so $u^2 - 7u + \frac{1}{2} = 0$

$u = \frac{7 \pm \sqrt{47}}{2}$

product of roots $= 7^7$ $\therefore$ $\boxed{B}$

(25) Solve system:

$x=3, y=4, z=7$

$\boxed{B}$

(26) for $n=1$, prob $= 1 - \frac{1000}{1000}$

$n=2$, prob $= 1 - \frac{1000}{1000} \cdot \frac{999}{1000}$

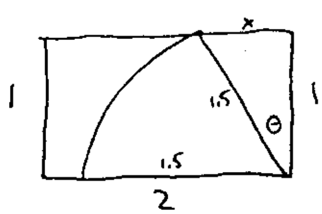
$\vdots$

$n=10$, prob $= 1 - \frac{1000}{1000} \cdot \frac{999}{1000} \cdots \frac{991}{1000}$

Find greatest n where prob $< .25$

$n = 24$ $\boxed{B}$

(27)



$x = \sqrt{1.5^2 - 1^2} = \sqrt{\frac{5}{2}}$

$\cos\theta = \frac{1}{1.5}$

$\frac{x}{2}$ = area of triangle

$\frac{(90-\theta)}{360} \cdot \pi \cdot 1.5^2$ = area of sector

Total area $\approx 1.4 \text{ ft}^2$ $\boxed{B}$

(28) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\boxed{C}$

(29) $v = r\omega$

$v = 5m \cdot \frac{57\text{ deg}}{\text{min}} \cdot \frac{100\text{ cm}}{1\text{ m}} \cdot \frac{1\text{ inch}}{2.54\text{ cm}} \cdot \frac{\pi\text{ rad}}{180\text{ deg}} \cdot \frac{1\text{ min}}{60\text{ sec}} = 3.26 \frac{\text{inch}}{\text{sec}}$

$\boxed{E}$

(30) $H(n) = \frac{n(n-1)}{2}$

$H(n+3) = \frac{(n+3)(n+2)}{2}$

$\frac{n^2+5n+6-n^2+n}{2} = 42$

$3n+3 = 42$

$n = 13$

$\boxed{D}$

Team cntd.

(4) $y = \frac{2x^2+10x+13}{x+1}$

slant asym

$$\begin{array}{r} 2x + 8 \\ x+1 \overline{) 2x^2 + 10x + 13} \\ \underline{2x^2 + 2x} \\ 8x + 13 \end{array}$$

horizontal asymptote

boundaries $\rightarrow$ $y = 2x+8$

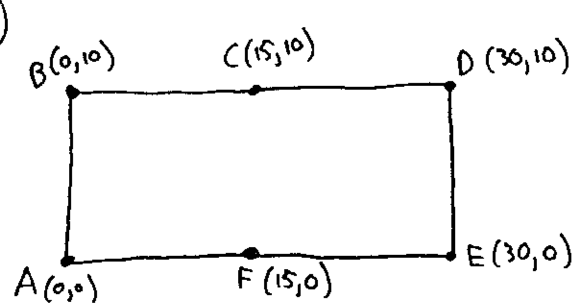
$x = -1$

$y = 0$

vertices at $(-1,0)$, $(-1,6)$, $(-4,0)$

$\frac{1}{2}bh = \frac{1}{2} \cdot 3 \cdot 6 = \boxed{9}$

(15)

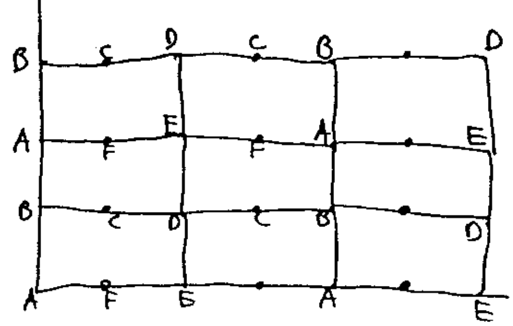


vertical lattice points spaced by 10

horizontal spaced by 15

Draw a line from origin to find first lattice point it touches

reflect the table

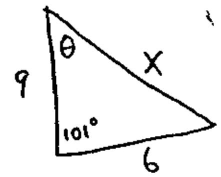


since slope $= \frac{4}{7}$, it will reach the point $(105, 60)$, which will be named

$\boxed{F}$

Pre-Calculus Team Solutions

1

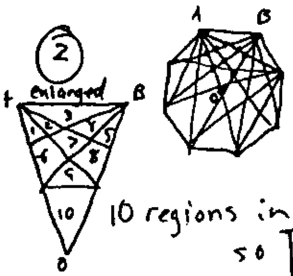


$x^2 = 9^2 + 6^2 - 2\cdot 9\cdot 6\cdot \cos 101°$

≈ 11.73

$\frac{x}{\sin 101°} = \frac{6}{\sin\theta}$ $\quad \theta \approx \boxed{30.1°}$

2



Take triangle ABO, then count the regions, and multiply by 8, since there are 8 triangles similar to ABO.

10 regions in ΔABO, so $\boxed{80}$ total

3 Use determinant method to find areas.

Largest → (3,6) (−1,−1) (3,−2)

$\frac{\begin{vmatrix} 3 & -1 & 3 \\ 6 & -1 & -2 \end{vmatrix}}{2} = \frac{|17+15|}{2} = 16$

Smallest → (0,4) (3,6) (−1,−1)

$\frac{\begin{vmatrix} 0 & 3 & -1 \\ 4 & 6 & -1 \end{vmatrix}}{2} = \frac{|-7-6|}{2} = 6.5$

Difference = $\boxed{9.5}$

14 $n = abbcd$

$n = b^2(10a+b)(10^2 d + 10d + b)$

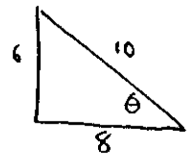
$b = 1, 3,$ or 7 – but the units digit of n is the units digit of b^4, so $d = 1$, and $b = 3$ or 7

since $(10^2 d + 10d + b)$ is prime, $b = 3$

with a little testing, $a = 2$, $c = 9$

$2+3+9+1 = \boxed{15}$

5



$\cos^{-1} .8 = \theta$ $\quad \frac{8}{\cos\frac{\theta}{2}}$ = longest bisector

$\sqrt{6^2+4^2}$ = shortest median

diff = $\boxed{1.22}$

6 $2^{1/3}\cdot 4^{1/9}\cdot 8^{1/27}\cdots$

$= 2^{1/3}\cdot 2^{2/9}\cdot 2^{3/27}\cdots$

infinite series $\frac{1}{3}+\frac{2}{9}+\frac{3}{27}\cdots = \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{\frac{1}{9}}{1-\frac{1}{3}}\cdots$

$= \frac{\frac{1}{2}}{1-\frac{1}{3}} = \frac{3}{4}$

$\boxed{2^{3/4}}$

7 $f(g(x)) = \sqrt{\text{Arcsin}(x-3)}$

$\text{Arcsin}(x-3) \geq 0$

$[3,4]$

$g(f(x)) = \text{Arcsin}(\sqrt{x}-3)$

$-1 \leq \sqrt{x}-3 \leq 1$

$[4,16]$

intersection $\boxed{x=4}$

8 $A = (-5+3)^8 = 256$

$B = {}_8C_4\cdot 1^4\cdot(-3)^4 = 5670$

$C = {}_4C_1\cdot 3^3\cdot 1^1 = 108$

$D = {}_5C_2\cdot 2^2\cdot\left(-\frac{1}{8}\right)^3 = -\frac{5}{64}$

$\boxed{21}$

9 A → odd # of factors if it is a perfect square, so 19 total

B → 12 integral factors

$12 = 12\cdot 1$
$\quad\; 6\cdot 2$
$\quad\; 4\cdot 3$

prime factored -- $x^{11}, x^5 y, x^3 y^2$

x^{11} → no factors

$x^5 y$ → $2^5\cdot 3, 2^5\cdot 5, 2^5\cdot 7, 2^5\cdot 11, 2^5\cdot 13, 3^5\cdot 2$

$x^3 y^2$ → $2^3\cdot 3^2, 2^3\cdot 5^2, 2^3\cdot 7^2, 3^3\cdot 2^2, 5^3\cdot 2^2$

11 total $\quad 11+19 = \boxed{30}$

10 A. period of $\sin^2(4x)$ is half of period of $\sin(4x) = \frac{\pi}{4}$

B. period of $|\sin(6x)|$ is half of period of $\sin(6x) = \frac{\pi}{6}$

C. period of $\sin(-4x) + \sin(6x)$ is period of $\sin(\text{GCF}(-4,6)x) =$

$\frac{C}{A+B} = \boxed{2.4}$

11 disregard height of bat and fence (they cancel each other)

$\Delta y = x\sin 35° t + \cancel{\;}-\cancel{\;}16t^2$

$\Delta x = x\cos 35° t$

$\Delta y = 0$, so $16t = x\sin 35°$

$\frac{x^2 \sin 35° \cos 35°}{16} = \Delta x = 2$

$x = 92.27$, so

$x = \boxed{93}$

13

$$\begin{bmatrix} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{bmatrix}$$

row reduces to

$$\begin{bmatrix} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{bmatrix}$$

$\boxed{h=5}$ or system is inconsistent

12 order the cards from 1-52, top to bottom

After 1 pass, all odds removed

After 2 passes, only #'s divisible by 4 remain

40th card is left.

This is the $\boxed{\text{Ace of Clubs}}$

~~4~~
~~8~~
~~12~~
~~16~~
~~20~~
~~24~~
~~28~~
~~32~~
~~36~~
40
~~44~~
~~48~~
~~52~~