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Calculus Individual Test FAMAT Invitational January 2001

1. d
2. c
3. b
4. b
5. a
6. d
7. b
8. c
9. c
10. e
11. c
12. c
13. a
14. d
15. b
16. d
17. b
18. d
19. b
20. d
21. c
22. a
23. b
24. d
25. c
26. d
27. b
28. b
29. b
30. c

FAMAT INVITATIONAL

Calculus Individual January Regionals

d 1. $f'(x)=3x^2-14x+3$ $f'(1)=-8$ $(1,-1)$ $m_\perp=\frac{1}{8}$

$x-8y=9$

c 2. $\frac{1}{\sqrt{1-(e^x)^2}}\cdot e^x=\frac{e^x}{\sqrt{1-e^{2x}}}$

b 3. $f''(3)<0$ $f(0)=0$ $f'(1)>0$

b 4.

a 5. $f'(x)=x\cdot e^{2-x}\cdot -1+e^{2-x}=e^{2-x}(-x+1)$ $\overset{f'}{\underset{1}{+\ \ -}}$

d 6. $f'(x)=\frac{1}{2}(1+2\sin x)^{-1/2}\cdot\cos x\cdot 2$ $f'(0)=1$ $(0,1)$

$x-y=-1$ $.03-y=-1$ $-y=-1.03$ $y=1.03$

b 7. $\frac{dr}{dt}=.02$ $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ $\frac{dV}{dt}=4\pi(15)^2(.02)=56.549$

c 8. $\frac{(2-\sec x)(-3\sin x)-3\cos x(-\sec x\tan x)}{(2-\sec x)^2}=\frac{-6\sin x+3\tan x+3\tan x}{(2-\sec x)^2}$

$\frac{6\tan x-6\sin x}{(2-\sec x)^2}=\frac{6(\tan x-\sin x)}{(2-\sec x)^2}$

c 9. $f'(x)=36x^5-40x^3$ $f''(x)=180x^4-120x^2=60x^2(3x^2-2)$

$+\ \ -\ \ -\ \ +$

$-\sqrt{2/3}\quad 0\quad \sqrt{2/3}$ ②

e 10. $\frac{1}{3}c^{-2/3}\frac{dc}{dt}=3\frac{dc}{dt}$ $c^{-2/3}=9$ $c=9^{-3/2}$ $c=\frac{1}{27}$ ⓔ

c 11. $\frac{1}{1+x^2}$ $f'(1)=\frac{1}{2}$

c 12. $H'(x)=2[f(x)]\cdot f'(x)$ $H''(x)=2[f(x)]f''(x)+2f'(x)f'(x)$

$H''(a)=2(+)(+)+2(-)(-)\therefore H''(a)>0$

a 13. $2x+xy'+y=0$ $y'=\frac{-y-2x}{x}$ $4+2y=6$ $2y=2$ $y=1$ $\frac{-1-4}{2}=-5/2$

d 14. 2 is not in the domain

b 15. $y'=\frac{-1}{x^2}$ $\frac{-1}{x^2}=-2$ $x^2=\frac{1}{2}$ $x=\pm\frac{1}{\sqrt{2}}$

d 16. 

$A(x)=2x(16-x^2)$ $A'(x)=32-6x^2$ $3x^2=16$ $x=\frac{4}{\sqrt{3}}$

$A(x)=32x-2x^3$ $0=16-3x^2$ $x^2=16/3$ $x=\frac{4\sqrt{3}}{3}$

$l=\frac{8\sqrt{3}}{3}$ $w=16-\frac{16}{3}=\frac{32}{3}$ $P=\frac{64}{3}+\frac{16\sqrt{3}}{3}$

b 17. $\lim_{x\to 2^-}x^2=4$ $\lim_{x\to 2^+}(8-2x)=4$ $f(2)=4$ $\therefore$ cont

not differentiable at $x=2$

d 18. $v(t)=4t+2\sin t$ $a(t)=4+2\cos(t)$ $a(\frac{\pi}{6})=4+2(\frac{\sqrt{3}}{2})$

b 19. $P(x)=50x-0.01x^2-(3000-20x+.03x^2)=70x-.04x^2-3000$

$P'(x)=70-.08x$ $P'(100)=70-8=62$

d 20. $f'(x) = 2^{\cot x} \cdot \ln 2 \cdot -\csc^2 x$ $\quad f'(\frac{\pi}{4}) = 2 \cdot \ln 2 \cdot (-2)$

$f'(\frac{\pi}{4}) = -4\ln 2$ $\quad m_{\perp} = \frac{1}{4\ln 2} = \frac{1}{\ln 16}$

c 21. $x\left[\frac{2}{5}(1-x)^{-3/5} \cdot -1\right] + (1-x)^{2/5} \cdot 1 = -\frac{2}{5}x(1-x)^{-3/5} + (1-x)^{2/5}$

$f'(x) = (1-x)^{-3/5}\left[-\frac{2}{5}x + 1 - x\right] = (1-x)^{-3/5}(1 - \frac{7}{5}x)$

$f''(x) = (1-x)^{-3/5} \cdot -\frac{7}{5} + (1 - \frac{7}{5}x)\left[-\frac{3}{5}(1-x)^{-8/5} \cdot -1\right]$

$f''(x) = -\frac{7}{5}(1-x)^{-3/5} + \frac{3}{5}(1 - \frac{7}{5}x)(1-x)^{-8/5}$

$f''(x) = -\frac{1}{5}(1-x)^{-8/5}\left[7(1-x) - 3(1 - \frac{7}{5}x)\right]$

$= -\frac{1}{5}(1-x)^{-8/5}\left[7 - 7x - 3 + \frac{21}{5}x\right] = -\frac{1}{5}(1-x)^{-8/5}\left[4 - \frac{14}{5}x\right]$

$\frac{14}{5}x = 4 \quad x = \frac{20}{14} = \frac{10}{7} \quad y = \frac{10}{7}(1 - \frac{10}{7})^{.4} \cong 1.02$

a 22. $\frac{ds}{dt} = 750 \text{ in}^2/\text{yr}$ $\quad S = \pi d h \quad S = 1200\pi d \quad \frac{dS}{dt} = 1200\pi \frac{dd}{dt}$

$-750 = 1200\pi \frac{dd}{dt} \quad \frac{dd}{dt} = \frac{-750}{1200\pi} = -.20$

b 23. $y' = 2\pi e^x$

d 24. $\frac{2M^2 - N}{1 + MN}$

c 25. by product rule $\frac{d}{dx}\left[x^2 \sin x\right] = x^2 \cos x + 2x \sin x \quad \therefore$

$\left.x^2 \sin x\right|_0^{\pi/4} = \frac{\pi^2}{16} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\pi^2}{32}$

d 26. $f'(x) = 2\sec^2(2x) \quad f'(\frac{\pi}{6}) = 2(2)^2 = 8 \quad f^{-1}(x) = \frac{\arctan(x)}{2}$

iii T $\quad$ IV T $\quad$ V f'' is odd

b 27. $\frac{t(e^t) - e^t}{t^2} = \frac{e^t(t-1)}{t^2}$ $\quad \frac{-\quad +}{1}$

b 28. $F'(x) = \sqrt{1 + 8\sin^3 x} \cdot 2\cos x \quad F'(\pi) = -2$

b 29. $\frac{\sqrt{2} - 0}{\sqrt{2}} = \frac{-2x}{2\sqrt{2 - x^2}} \quad 1 = \frac{-x}{\sqrt{2 - x^2}} \quad \sqrt{2 - x^2} = -x \quad 2 - x^2 = x^2 \quad 2 = 2x^2 \quad x^2 = 1 \quad x = \pm 1 \quad \therefore -1$

c 30. $y' = 3x^2 \quad m_{norm} = \frac{-1}{3x^2}$ $\quad$ for c $y = \frac{1}{3}x^{-1} \quad y' = -\frac{1}{3}x^{-2}$