Precalculus Team #9.

For how many integral values of x is f(x) an integer?

$f(x)=\dfrac{36+12x}{x^2+8x+15}=\dfrac{12(3+x)}{(x+5)(x+3)}=\dfrac{12}{x+5}$ if $x \neq -3$. Setting $x+5=$ all possible integral factors of 12.

$x+5=\pm 1$	$x+5=\pm 2$	$x+5=3$	$x+5=\pm 4$	$x+5=\pm 6$	$x+5=\pm 12$
$x=-6,\ -4$	$x=-7,\ 3$	$x=-2$	$x=-9,\ -1$	$x=-11,\ 1$	$x=7,\ -7$

Precalculus Team #13

Solve for x if $0 \le x < 2\pi$ and $3\cos^2 x\ \sin x - \sin^3 x = 1$.

$$\sin x(3\cos^2 x-(1-\cos^2 x))=1$$
$$\sin x(4\cos^2 x-1)=1$$
$$\sin x(4(1-\sin^2 x)-1)=1$$
$$\sin x(4-4\sin^2 x-1)=1$$
$$3\sin x-4\sin^3 x-1=0$$
$$4\sin^3 x-3\sin x+1=0 \qquad \sin x=-1 \text{ is a solution.}$$

$$\begin{array}{r|rrrr} -1) & 4 & 0 & -3 & 1 \\ & & -4 & 4 & -1 \\ \hline & 4 & -4 & 1 & 0 \end{array}$$

$4\sin^2 x-4\sin x+1=0 \qquad (2\sin x-1)^2=0 \qquad \sin x=\dfrac{1}{2}$

Answer : $\dfrac{3\pi}{2}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Precalculus Team #11

Let $a_1, a_2, a_3\ a_4, \ldots\ldots$ be an arithmetic progression. If $a_r = s^2$ and $a_s = r^2$, where r and s are distinct positive integers, then find a_{r+2} in terms of r and s.

Answer: $s^2-2s-2r$

Let d be the common difference between consecutive terms of this arithmetic progression.

Thus $a_r = a_1+(r-1)d$ and $a_s = a_1+(s-1)d$.

Since $s^2-r^2=a_r-a_s=(r-s)d$, we see that $d=\dfrac{s^2-r^2}{r-s}=-(r+s)$.

Therefore $a_{r+2}=a_r+2d=s^2-2(r+s)=s^2-2r-2s=s^2-2s-2r$.

Precalulus Team #12

Answer: $P=\dfrac{5}{3};\ Q=\dfrac{1}{3}$

If P and Q are constants such that $\dfrac{P}{e^x-1}+\dfrac{Q}{e^x+2}=\dfrac{2e^x+3}{e^{2x}+e^x-2}$ Find P and Q.

$\therefore P(e^x+2)+Q(e^x-1)=2e^x+3$ and $Pe^x+2P+Qe^x-Q=2e^x+3$

$\therefore Pe^x+Qe^x=2e^x$ and $P+Q=2$

$2P-Q=3$ so $3P=5 \qquad P=\dfrac{5}{3}$ and $\dfrac{5}{3}+Q=2$ *so* $Q=\dfrac{1}{3}$

Pre Calculus
Solution #10

-121 1/4 #10

$x^2+y^2+10y+25=7+25$

$x^2+(y+5)^2=32$ is

a circle with center $(0,-5)$ and radius $4\sqrt{2}$

Since the radius is $\perp$ to the line

$y=-x+3$ It has slope 1

and equation $-5=0+b$ $\quad y=x-5$

$-5=b$ $\quad y=-x+3$

$(h,k)=(4,-1)$ $\quad 2y=-2$

$(m,0)=-4$ $f'(x)=-12x^2+3$ $\quad y=-1$

-4 $\quad -48+3$ $\quad -1=x-5$

$(-45,0)$ $\quad -45$ $\quad 4=x$

(t,v) $\quad (a,b)=\frac{-b}{2a}=\frac{2}{2}=1$

(ab) (b,c) $\quad (1,2)$ $\quad b=1^2-2(1)+3$

$(1,2)$ $(2,6)$ $\quad 1-\frac{2}{2}+3$

(t,v) $\left(\frac{3}{2},4\right)$ = midpoint $\quad \frac{4}{c}=\frac{c}{9}$ $\quad c^2=36$

$c=6$

$$\frac{1}{2}\begin{vmatrix} 4 & -1 & 1 \\ -45 & 0 & 1 \\ \frac{3}{2} & 4 & 1 \end{vmatrix}\begin{matrix} 4 & -1 \\ -45 & 0 \\ 3/2 & 4 \end{matrix} = \left(0-\frac{3}{2}-180\right) - (0+16+45)$$

$(-181\frac{1}{2}-61)$

$-181\frac{1}{2}$ $\quad \frac{1}{2}\left(-\frac{363}{2}-\frac{122}{2}\right)$

$\frac{363}{2}$ $\quad -\frac{485}{4}=121\frac{1}{4}$

-5 #7 $\sum_{k=1}^{31} \log_2\left(\frac{k}{k+1}\right) =$ Pre Calculus Team Solutions

$(\log_2 1 + \log_2 2 \ldots \log_2 31) - (\log_2 2 - \log_2 3 \ldots \log_2 32$
$\log_2 1 - \log_2 32$
$0 \quad -5 = -5$

Pre Calculus Team #10

8 $4x^3 + kx^2 - 9x - 18 = 0$
$4x^2(x + \frac{k}{4}) - 9(x+2) = 0$
$(4x^2 - 9)(x + \ ) = 0$
therefore $\frac{k}{4} = 2$
$k = 8$

\# 14 $1 + \frac{1}{x} = x$
$x + 1 = x^2$
$x^2 - x - 1 = 0$ $\quad \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$\frac{1 \pm \sqrt{5}}{2}$

15 $128^{(\pi + \frac{4}{7})} \cdot 8^{(\frac{n}{3} - \frac{7\pi}{3})} = 64^{(\frac{n}{2} - \frac{1}{3})}$
3 $2^{7(\pi + \frac{4}{7})} \cdot 2^{3(\frac{n}{3} - \frac{7\pi}{3})} = 2^{6(\frac{n}{2} - \frac{1}{3})}$
$2^{7\pi + 4} \cdot 2^{n - 7\pi} = 2^{3n - 2}$
$(7\pi + 4) + (n - 7\pi) = 3n - 2$
$4 + n = 3n - 2$
$6 = 2n$
$3 = n$

Precalculus team solutions

1. In triangle ABC, $a = 3\sqrt{2}$ $b = 4\sqrt{2}$ $A = 45°$, and B is obtuse. Find the value or possible values of c.

$4-\sqrt{2}$; From the law of cosines, we see that

$$18 = (3\sqrt{2})^2 = (4\sqrt{2})^2 + c^2 - 2(4\sqrt{2})c[\frac{\sqrt{2}}{2}] = 32 + c^2 - 8c.$$

Since $c^2 - 8c + 14 = 0$, $c = 4 \pm \sqrt{2}$. Since $A = 45°$ and B is obtuse, C must bethe smallest angle in the triangle. *Hence c must be the smallest side of the triangle. Since* $4+\sqrt{2} > 3\sqrt{2}$, $c = 4-\sqrt{2}$

2 Find the polar coordinates of the points satisfying both polar equations. $0 \le \theta < 360$.

$r = 4\sin\theta$ $r = 4\cos 2\theta$; $4\sin\theta = 4\cos 2\theta$

$$4\sin\theta = 4(1-2\sin^2\theta)$$

$$\sin\theta = (1-2\sin^2\theta)$$

$$\sin^2\theta + \sin\theta - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$\sin\theta = \frac{1}{2}$; $\sin\theta = 1$; $\therefore (2,\ 30°)$; $(2,\ 150°)$; $(-4,\ 270°)$.

3. A bag contains 7 balls of 7 different colors. One is black and one is white. If 3 balls are selected without returning them to the bag, what is the probability that the seletion contains either the white ball or the black ball?

$P(w, Nb, Nb) = 3(\frac{1}{7} \bullet \frac{5}{6} \bullet \frac{4}{5})$; *any order*

$P(b,\ Nw,\ Nw) =$ same; *any order.* $\therefore P(w \text{ or } b, \text{ but not both}) = 2[3 \bullet \frac{1}{7} \bullet \frac{5}{6} \bullet \frac{4}{5}] = \frac{4}{7}$

4. *A student and her teacher completed a certain task in two days. They completed three - fifths of the task* the first day with the student working six hours and the teacher working twenty hours. On the next day they finished the remaining two - fifths of the task with the student working three hours and the teacher working fifteen hours. How many hours would it take the student working alone, to complete this task?

student :30; teacher: 50.

Let x be the number of hours that it would take the student, working alone, to complete this task and let y be the number of hours that it would take the teacher, working alone, to complete this task. Since they completed three - fifths of the task the first day with the student working six hours and the teachre working twenty hours, we see that $\frac{6}{x} + \frac{20}{y} = \frac{3}{5}$. Since they finish two - fifths of the task with the student working three hours and the teacher working fifteen hours, we see that $\frac{3}{x} + \frac{15}{y} = \frac{2}{5}$. Multiplying the second equation by 2 and then subtracting the first equation from this new equation gives us. $\frac{30}{y} - \frac{20}{y} = \frac{1}{5}$ *thus* $\frac{10}{y} = \frac{1}{5}$ *and* $y = 50$. Thus x =30.

Pre Calculus Team #5

-4, -5, -6 Let $a = x^2+10x+30$

$$(x^2+10x+30)^2 = 11x^2+110x+330-30$$

$$(x^2+10x+30)^2 = 11(x^2+10x+30)-30$$

$$a^2 = 11a-30$$

$$a^2-11a+30=0$$

$$(a-6)(a-5)=0$$

$$a=6 \text{ or } a=5$$

$x^2+10x+30=6$ $\quad$ $x^2+10x+30=5$

$x^2+10x+24=0$ $\quad$ $x^2+10x+25=0$

$(x+4)(x+6)=0$ $\quad$ $(x+5)^2=0$

$x=-4,-5,-6$

105 Team #6 There are four ways to choose the final integer times

Consider the following mutually exclusive possibilities

0 is the digit in the units place

0 is not the digit in the units place

If 0 is the digit in units, only one choice for digit in units place, 6 choices for digit in hundreds place and 5 choices for digit in 10's place

If zero not in units there are 3 choices there (2,4,6) five digits for hundreds place and 5 choices for digit in units place

$$1 \cdot 6 \cdot 5 + 3 \cdot 5 \cdot 5 = 30 + 75 = 105$$