

Lincoln Park Math Invitational

March 23, 2002

Precalculus Individual

Choose the best answer for each of the following. If the answer is not found among the choices, choose "e" for none of the above., You will receive 4 pts. for each correct response and -1 pt. for each incorrect response.

- If A and B are intersecting sets and A' is the complement of A, then $(A \cap B) \cup (A' \cap B) =$

a. A' b. B' c. B d. $(A \cap B')$ e. nota
- Let a and b be real numbers such that $a + bi = (6 + 9i + 8i^2)(7 - 3i^5 + 9i^{11})$.
The value of $|a| + |b|$ is:

a. -57 b. -55 c. 181 d. 119 e. nota
- Let $g(x) = \frac{2x+5}{x+1}$ and $f(x) = \log_2\left(\frac{x^3+x^2+x+25}{2x-4}\right)$. Find $(f \circ g)(2)$.

a. -5 b. 5 c. undefined d. 32 e. nota
- Evaluate: $\cos(\text{Arc tan}(-\frac{3}{4}) + \text{Arc sin}(\frac{5}{13}))$.

a. $\frac{16}{65}$ b. $\frac{33}{65}$ c. $\frac{56}{65}$ d. $\frac{63}{65}$ e. nota
- The distance between the centers of the circles: $x^2 - 4x + y^2 + 2y = 4$ and $x^2 + y^2 - 6y = 0$ is

a. $\sqrt{5}$ b. $2\sqrt{5}$ c. $2\sqrt{2}$ d. $\sqrt{2}$ e. nota
- If a person is dealt two cards from a 52 - card deck, how many different hands are possible if order is not important?

a. 13 b. 103 c. 1326 d. 2652 e. nota
- If $3 + 2i$ and $2 - 3i$ are zeros of $P(x) = x^5 - 12x^4 + 70x^3 - 230x^2 + 429x - 338$.
which of the following is NOT a zero?

a. $3 - 2i$ b. 2 c. $2 + 3i$ d. 3 e. nota
- If x is in Quadrant II and $\tan x = \frac{-21}{20}$, then $\sin(x + 45^\circ)$ equals

a. $\frac{-\sqrt{3}}{58}$ b. $\frac{\sqrt{3}}{29}$ c. $-\frac{\sqrt{2}}{58}$ d. $-\frac{\sqrt{2}}{29}$ e. nota

9. Find the sixth term of the expansion of $\left(\begin{matrix} 2x^2 & y^3 \\ -1 & 1 \end{matrix} \right)^7$

a. $21x^2y^5$

b. $84x^4y^{15}$

c. $7x^3y^4$

d. $56x^6y^{12}$

e. *nota*

10. Find $\lim_{x \rightarrow 1} \left(\frac{2x^3 + 3x^2 - 2x - 3}{x^2 - 1} \right)$

a. nonexistent

b. $-\frac{3}{2}$

c. -1

d. 5

e. *nota*

11. Which of the following vectors is perpendicular to the vectors $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$?

a. $\begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$

b. $\begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$

c. $\begin{pmatrix} -10 \\ 2 \\ -14 \end{pmatrix}$

d. $\begin{pmatrix} -9 \\ -10 \\ 12 \end{pmatrix}$

e. *nota*

12. If (a, b) and (c, d) are on the line with equation $y = mx + k$, then the distance between (a, b) and (c, d) in terms of a, c and m is:

a. $\frac{|a - c|}{\sqrt{1 + m^2}}$

b. $|a - c| |m|$

c. $\frac{|a - c|}{|m|}$

d. $|a - c| \sqrt{1 + m^2}$

e. *nota*

13. The area bounded by the curve $y = \sqrt{16 - x^2}$ and the x-axis is

a. 4π

b. 8π

c. 16π

d. 32π

e. *not*

14. If $f(x) = \begin{cases} x^2 & x \leq 1 \\ 2x - 1 & x > 1 \end{cases}$ then

a. $f(x)$ is not continuous at $x = 1$

b. $f(x)$ is continuous at $x = 1$ but $f'(x)$ does not exist

c. $f'(x)$ exists and equals one

d. $f'(1) = 2$

e. *nota*

15. The sum of the first eleven terms of the sequence $\left\{ (-1)^n \cdot \cos \frac{3\pi}{2} \right\}$ is

a. -1

b. 0

c. 1

d. 11

e. *nota*

16. If x and y are real numbers, then the domain of the function $f(x) = \sqrt{9-x^2}$ is:
- a. $|x| \geq 3$ b. $|x| \leq 3$ c. all x except 3 or -3 d. all real numbers e. nota

17. Out of all five digit numbers whose sum of the digits is 43, what is the probability that the number is divisible by 11?

- a. $\frac{1}{5}$ b. $\frac{3}{5}$ c. $\frac{17}{56}$ d. $\frac{218}{239}$ e. nota

18. Find the 66th derivative of $\cos x$.

- a. $\cos x$ b. $-\cos x$ c. $\sin x$ d. $-\sin x$ e. nota

19. Which of the following is a circle with the center at $(-1, 5)$ and tangent to the line $6x + 8y = 14$

- a. $(x+1)^2 + (y-5)^2 = 19$ b. $(x+1)^2 + (y-5)^2 = 12$ c. $(x+1)^2 + (y-5)^2 = 4$
d. $(x+1)^2 + (y-5)^2 = 2$ e. nota

20. Solve for $x > 0$: $(e^{\ln(x-1)})(\ln e^{x+5}) = 3 \ln e + e^{2 \ln 2}$

- a. 2 b. 3 c. 4 d. 6 e. nota

21. What is the distance between parallel lines $y = 2x + 5$ and $y = 2x + 1$?

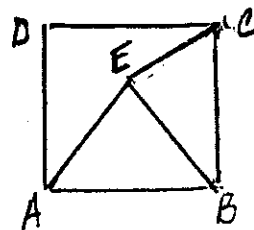
- a. $\frac{4\sqrt{5}}{5}$ b. $\frac{5\sqrt{5}}{4}$ c. 2 d. 4 e. nota

22. What is the equation of the locus of points equidistant from $A(-2, 1)$ and $B(3, 4)$?

- a. $5x + 3y = 15$ b. $5x + 3y = 10$ c. $x + 5y = 10$ d. $10x + 6y = 25$ e. nota

23. ABCD is a square with side 4. Triangle ABE is equilateral. Find EC.

- a. $4\sqrt{2-\sqrt{3}}$ b. $4-2\sqrt{3}$ c. 2
d. $16(2-\sqrt{3})$ e. nota



24. Simplify and combine like terms. $\sqrt{54x^5} + \sqrt[3]{36x^6}$

- a. $\sqrt{54x^5} + \sqrt[3]{36x^6}$ b. $(3x^2 + x)\sqrt{6x}$ c. $3x^2\sqrt{x} + \sqrt[3]{36x^6}$ d. $3x^2(\sqrt{6x} + 2)$ e. nota

25. The focus of the parabola $y = \frac{1}{6}x^2$ is

- a. (0, 6) b. (0, $\sqrt{6}$) c. (0, $\frac{3}{2}$) d. (0, 0) e. nota

26. If $f(x) = \tan x$, evaluate $f^{-1}(f(\frac{3\pi}{4}))$

- a. $\frac{\pi}{4}$ b. $\frac{-\pi}{4}$ c. $\frac{3\pi}{4}$ d. $\frac{-3\pi}{4}$ e. *nota*

27. Simplify where the domain applies :

$$\frac{\tan^3 x \cos x}{\sec^2 x} + \frac{1 + \cot^2 x}{\sec^2 x} + \frac{2 + \tan^2 x}{1 - \sec^4 x}$$

- a. $\tan^2 x$ b. $\cos^2 x + 1$ c. -1 d. $\sin^3 x$ e. *nota*

28. Find the sum of the squares of the possible values of x given :

$$(6x^2 + 2x - 21)^{(3x^2 - 13x - 10)} = 1$$

- a. $\frac{229}{9}$ b. $\frac{296}{9}$ c. 25 d. $\frac{119}{3}$ e. *nota*

29. The equation $2x^2 + \sqrt{3}xy + y^2 - 3 = 0$ is to be transformed by a rotation of axes to eliminate the xy term. What is the smallest counter clockwise rotation that will accomplish this?

- a. 30° b. 45° c. 60° d. 90° e. *nota*

30. Solve for x : $2^{x+1} = 7^{x+2}$

- a. $x = \frac{\ln(2/49)}{\ln(7/2)}$ b. $x = \frac{\ln(49/2)}{\ln(2/7)}$ c. $x = \ln(49/2) - \ln(2/7)$
d. $x = \ln(2/49) - \ln(7/2)$ e. *nota*